Date |
Page |

CHAPTER 2

Capacitors:-

Capacitors:-

↳ Capacitor is an electronic device which is used to store charge.

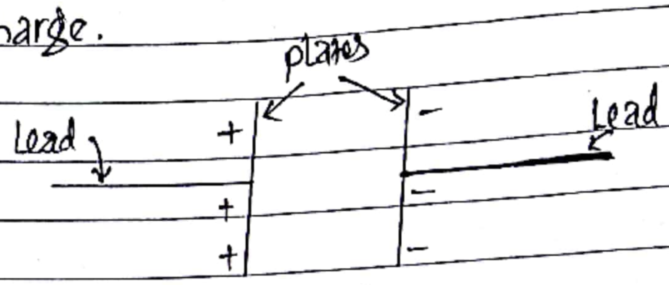


fig:- capacitor.

The simplest form of capacitor has two parallel conducting plates. There is dielectric medium such as air, paper, plastic film etc between two plates.

Capacitance of Capacitor:-

↳ The ability of a capacitor to store charge is known as it's capacitance.

Experimentally, It has been found that charge(q) stored in capacitor is directly proportional to p.d(V) across the plates of the capacitor.

i.e., $q \propto V$

$\Rightarrow \boxed{q = CV}$

Where, C is proportionality constant called capacitance of the capacitor. It's value depends upon shape and size of plate, their separation and the nature of dielectric medium.

Also, $C = \frac{q}{V}$

If $V = 1$ Volt. Then, $C = q$

Thus, Capacitance of capacitor is numerically equal to the electric charge required to raise p.d. by 1 volt.

It's SI unit is C/V or Farad (F).

Note: $\Rightarrow 1\ pF = 1\ \mu\mu F = 10^{-12}\ F$

Date | Page

Principle of a Capacitor:-

1) Since, the Capacitance of a Capacitor is given by:

$$C = Q/V$$

For constant charge 'Q', we can write

$$\Rightarrow C \propto \frac{1}{V}$$

If the electric potential of a Capacitor 'V' decreases the Capacitance 'C' of the capacitor increases or vice-versa. This is the working principle of the capacitor.

Explanation:-

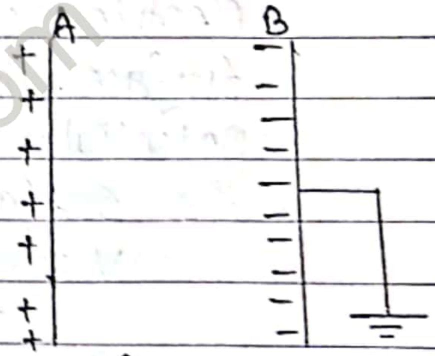


fig:- Capacitor

1) Suppose A and B are two metal plates very close to each other. The plate 'A' is positively charge untill it's electric potential becomes maximum. Due to it's electrostatic induction, negative charge is induced on the near surface and positive charge is induced on the outer surface of plate 'B'. The negative charge of plate 'B' tries to decrease the electric potential of plate 'A' while positive charge of plate 'B' tries to increase it.

As the negative charge being nearer to plate 'A', the resultant electric potential of plate 'A' decreases. Further when the plate 'B' is connected to earth, ~~so that the~~ induce positive charge escapes to earth so that the electric potential of plate 'A' decreases more due to presence of induced negative charge.

In the above arrangement, we see that the potential of plate 'A' always decreases such that it's capacitance

Date
Page

increases. In other word reduction in electric potential increases the Capacitance of the Capacitor by a large amount. This is the principle of capacitor.

Different types of Capacitor:-

(i) Isolated Spherical capacitors:-

↳ Let us Consider an isolated sphere of radius 'r' having electric charge 'q' on its surface. Then, the electric potential at any point on it's surface is:

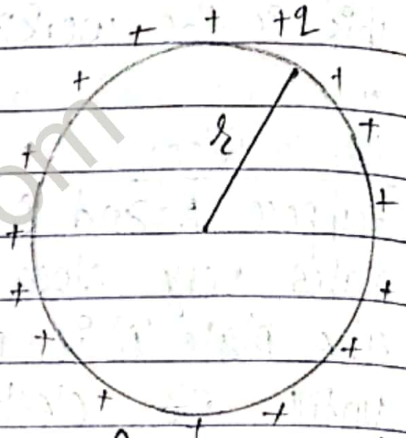


fig:- Isolated spherical Capacitor

$$V = \frac{q}{4\pi\epsilon_0 r}$$

$$\Rightarrow 4\pi\epsilon_0 r = q/V \text{ ---- (i)}$$

If C is the Capacitance of the sphere,

Then, $C = \frac{q}{V}$ ---- (ii)

Comparing eq^n(i) & (ii);

$$\Rightarrow \boxed{C = 4\pi\epsilon_0 r} \text{ ---- (iii)}$$

This is required expression for Capacitance of isolated spherical Capacitor. eq^n (iii) shows that Larger sphere Store more electric charge.

In CGS system;

$$\boxed{4\pi\epsilon_0 = 1}$$

$$\therefore \boxed{C = r} \text{ ---- (iv)}$$

Thus, in C.G.S. system Capacitance of a Charged sphere is numerically equal to the radius of sphere.

v.v.imp (ii) <u>parallel plate capacitor:-</u>

↳ parallel plate capacitor consists of two parallel plates each of equal area and placed very close to each other.

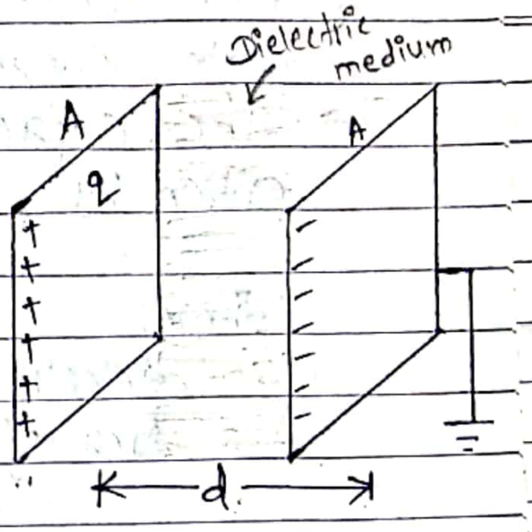


fig:- parallel plate capacitor

Let us consider two parallel plates having area 'A' of each and separated by distance 'd'.

Let '+q' be the electric charge given to one plate while the other plate is connected to ground.

Now, using Gauss's Theorem of electrostatic, the electric field intensity between two plate is given by:

$$E = \frac{\sigma}{\epsilon_0}$$

$$\Rightarrow E = \frac{q}{\epsilon_0 A} \text{ ---(i)} \quad [\because \sigma = \frac{q}{A}, \text{ is surface charge density}]$$

Also, let 'V' be the potential difference between two plate. Then, electric field intensity be,

$$E = \frac{V}{d} \text{ ---(ii)}$$

$$\frac{V}{d} = \frac{q}{\epsilon_0 A} \quad [\because \text{From (i) \& (ii)}]$$

or, $$\frac{q}{V} = \frac{\epsilon_0 A}{d} \text{ ---(iii)}$$

If 'C' be the capacitance of parallel plate capacitor. Then,

$$C = \frac{q}{V} \text{ ---(iv)}$$

Using eq^n (iv) in eq^n (iii), we get:

$$\Rightarrow \boxed{C = \frac{\epsilon_0 A}{d}} \text{ ---(V)}$$

This is required expression for capacitance of parallel plate capacitor if air of vacuum is placed between two plates.

If dielectric medium of dielectric constant 'k' placed between two plates of parallel plate capacitor Then,

$$\boxed{C = \frac{K \epsilon_0 A}{d}} \text{ ---(vi)}$$

5Q) How would you increase the capacitance of parallel plate capacitor?

↳ The capacitance of parallel plate capacitor is given by,

$$C = \frac{K \epsilon_0 A}{d} \quad \text{---(i)}$$

Where,
K = dielectric constant, ϵ_0 = permittivity of free space.
A = Area of each plate, d = separation between two plates.

From eqn(i), we can say that capacitance of parallel plate capacitor can be increased by;
i) increasing the area of plates
ii) decreasing the separation between two plates
iii) placing the medium between two plates having large dielectric constant

(iii) Spherical Capacitor:-

↳ The spherical capacitor consists of two sphere having radius 'a' and 'b' and separated by certain distance as shown in figure. Also let, +q charge is given to the inner sphere while the ~~other~~ outer sphere is connected to the earth.

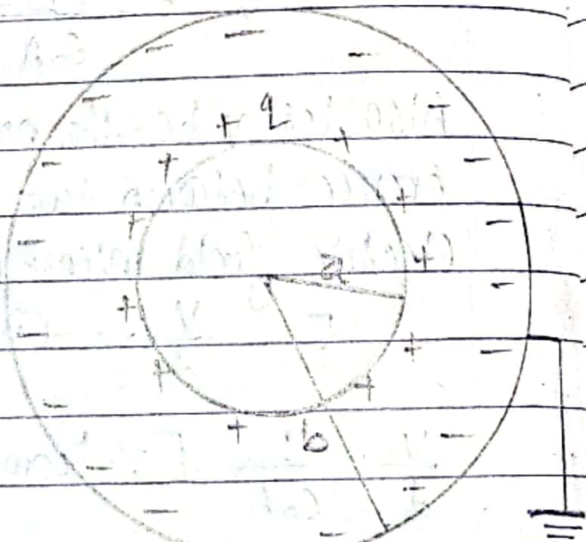


fig:- Spherical capacitor

Now, the electric potential at any point on the surface of inner sphere is,

$$V_a = \frac{q}{4\pi\epsilon_0 a} \quad \text{-----(i)}$$

and the electric potential at any point on the surface of outer sphere is;

$$V_b = \frac{-q}{4\pi\epsilon_0 b} \quad \text{---(ii)}$$

Therefore, the total potential of the system be,

$$V = V_a + V_b$$

$$\text{or, } V = \frac{q}{4\pi\epsilon_0 a} - \frac{q}{4\pi\epsilon_0 b}$$

$$\text{or, } V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]$$

$$\Rightarrow \frac{q}{V} = 4\pi\epsilon_0\left(\frac{ab}{b-a}\right) \quad \text{---(iii)}$$

If C be the Capacitance of Spherical Capacitor. Then,

$$C = \frac{q}{V} \quad \text{---(iv)}$$

Using; eq^n (iv) in eq^n (iii)

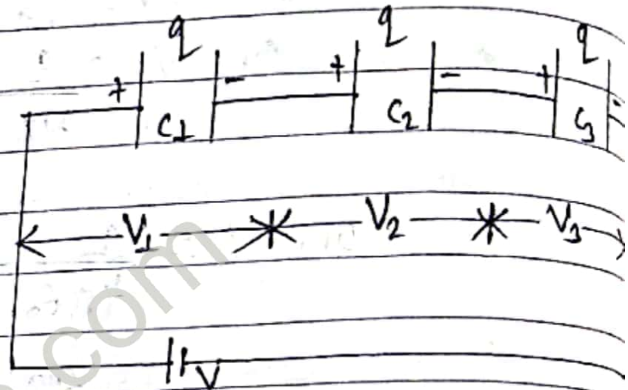
$$\Rightarrow \boxed{C = 4\pi\epsilon_0\left[\frac{ab}{b-a}\right]} \quad \text{---(v)}$$

This is required expression for Capacitance of Spherical Capacitor.

Combination of Capacitors:-

(i) Series Combination of Capacitors:-

↳ Capacitors are said to be in series combination if second plate of first capacitor is connected with first plate of second capacitor and so on. In series combination, electric charge on each plate is same while the voltage across them is different.



Let us consider three capacitors of capacitance C_1, C_2 & C_3 are connected in series across a battery of p.d. (V). Also, let 'q' be the charge on each capacitor and V_1, V_2 & V_3 are the p.d. across C_1, C_2 & C_3 respectively as shown in figure.

fig:- Series Combination of Capacitors

Then,

$$V_1 = \frac{q}{C_1} \qquad V_2 = \frac{q}{C_2} \qquad V_3 = \frac{q}{C_3} \qquad [\because q = CV]$$

Thus, the total voltage across them be,

$$V = V_1 + V_2 + V_3$$

$$\text{or, } V = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3} \qquad \Rightarrow \frac{V}{q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \text{ ---- (i)}$$

If C_s be the total capacitance of circuit.

Then, $C_s = q/V$

$$\Rightarrow \frac{1}{C_s} = \frac{V}{q} \text{ ---- (ii)}$$

Comparing eqn (i) & (ii):

$$\boxed{\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} \text{ ---- (iii)}$$

Thus, in series combination of capacitors the reciprocal of total capacitance is equal to the sum of reciprocal of all individual capacitance.

(ii) parallel Combination of Capacitors:-

↳ capacitors are said to be in parallel Combination if one side of plates are connected at one point and other side of plates are connected at other point. In parallel combination, Voltage across all capacitors are same while the electric Charge across them is different.

Let us consider three Capacitors of Capacitance C_1, C_2 and C_3 are Connect in parallel across a battery of p.d.(V). Also, Let 'V' be the p.d. across each Capacitor and q_1, q_2 and q_3 are the Charge across C_1, C_2 & C_3 respectively as shown in figure.

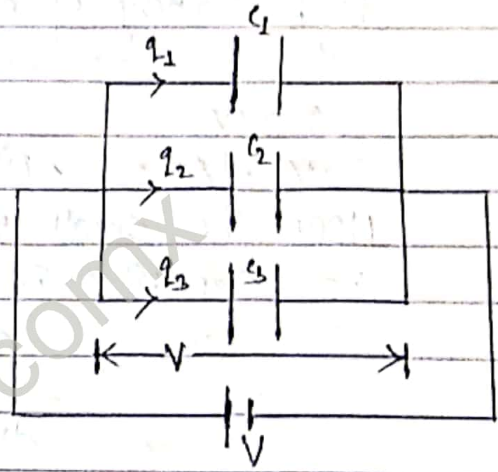


fig:- parallel Combination of Capacitors.

Then,

$q_1 = C_1 V$ $\quad$ $q_2 = C_2 V$ $\quad$ $q_3 = C_3 V$ $\quad$ $[\because q = CV]$

Thus, the total electric charge across the circuit;

$$q = q_1 + q_2 + q_3$$

$$\text{or, } q = C_1 V + C_2 V + C_3 V$$

$$\text{or, } \frac{q}{V} = C_1 + C_2 + C_3 \text{ -------- (i)}$$

If C_p be the total capacitance of the circuit.

Then, $C_p = \frac{q}{V}$ --- (ii)

Comparing eq^n(i) and eq^n(ii); we get;

$$\boxed{C_p = C_1 + C_2 + C_3} \text{ --- (iii)}$$

Thus, in parallel combination of Capacitors the total Capacitance of Circuit is equal to the sum of all individual Capacitance.

Energy Store in capacitors:-

↳ Let us consider a capacitor of capacitance 'c', which is initially chargeless but after some time it has charge 'q' when connected with battery of p.d.(V).

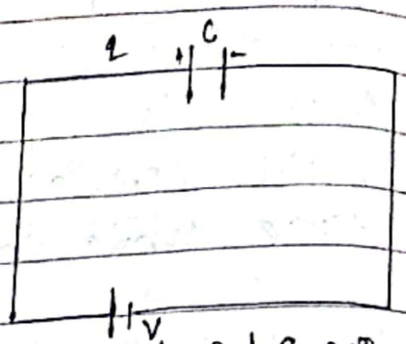


fig:- Charged Capacitor

Then,

$q = CV$

$V = q/c$ --- (i)

Now, the small work done(dW), when small charge dq stored in capacitor is given by;

$dW = V.\,dq$ $\quad [\because V = \frac{W}{q}]$

$dW = \frac{q}{c}.\,dq$ --- (ii) $\quad [\because$ Using eqn(i)$]$

So, total amount of work done in the capacitor on storing charge 'q' is given by;

$W = \int_0^q dW$

$= \frac{1}{C}\int_0^q q\,dq$

$= \frac{1}{C}\left[\frac{q^2}{2}\right]_0^q$

$= \frac{1}{2C}[q^2 - 0]$

$\therefore \boxed{W = \frac{q^2}{2C}}$ --- (iii)

Also, from eqn(i), $q = CV$

$\therefore W = \frac{1}{2C}C^2V^2$

$\Rightarrow \boxed{W = \frac{1}{2}CV^2}$ --- (iv)

Also, $C = q/V$

$\therefore W = \frac{1}{2}\frac{q}{V}.V^2$

$\Rightarrow \boxed{W = \frac{q.V}{2}}$ --- (v)

This work done is stored as energy on the charged capacitor. Thus, the energy stored on the charged capacitor is given by;

$\boxed{E = \frac{q^2}{2C} = \frac{1}{2}CV^2 = \frac{qV}{2}}$ --- (vi)

Energy density:-

↳ The energy stored per unit volume of the Capacitor is Called the energy density.

i.e., $\text{Energy density } (u) = \dfrac{\text{Energy Stored (E)}}{\text{Volume of Capacitor (V)}}$ ---- (i)

Now, the energy stored in the capacitor is given by,

$$E = \frac{1}{2}CV^2 \text{ ---- (ii)}$$

Where, C = capacitance of Capacitor & V = p.d. across the plate of Capacitor.

Let us Consider a parallel plate capacitor each plate of area 'A' and separated by distance 'd'. Then the volume of Capacitor is given by,

$$V = A \cdot d \text{ ---- (iii)}$$

Using eq^n (ii) & (iii) in eq^n(i);

$$\therefore u = \frac{\frac{1}{2}CV^2}{Ad}$$

Since, $C = \dfrac{\epsilon_0 A}{d}$ is capacitance of parallel plate capacitor.

$$\therefore u = \frac{1}{2}\,\frac{\frac{\epsilon_0 \cancel{A}}{d}V^2}{\cancel{A}d}$$

$$\text{or, } u = \frac{1}{2}\epsilon_0 \frac{V^2}{d^2}$$

$$\text{or, } u = \frac{1}{2}\epsilon_0 \left(\frac{V}{d}\right)^2$$

$$\Rightarrow \boxed{u = \frac{1}{2}\epsilon_0 E^2} \text{ ----- (iv)}$$

This is required relation for energy density and this relation shows that energy density is directly proportional to the square of electric field intensity.

Loss of energy:-

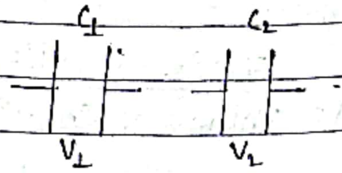


fig:- Before Connection

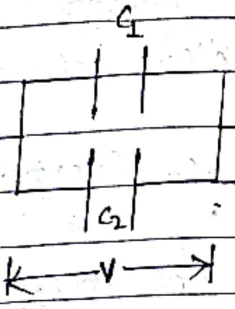


fig: After Connection

Energy before Connection;

$$\boxed{E_1 = \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2}$$

Energy after Connection

$$E_2 = \frac{1}{2} C_1 V^2 + \frac{1}{2} C_2 V^2$$

$$\boxed{E_2 = \frac{1}{2}(C_1 + C_2) V^2}$$

Where, V = Common potential

or, $\frac{\text{Total charge}}{\text{total capacitance}} = \frac{q_1 + q_2}{C_1 + C_2} = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$

$$\therefore E_2 = \frac{1}{2}(C_1 + C_2) \cdot \left[\frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}\right]^2$$

$$\Rightarrow \boxed{E_2 = \frac{1}{2} \frac{(C_1 V_1 + C_2 V_2)^2}{C_1 + C_2}}$$

Now, We Can easily Calculate,

$\boxed{\text{Loss of energy} = E_1 - E_2}$ #

Dielectric Substance:-

↳ The substance which do not have free electric charge but can transmitted electric field to it is called dielectric substance.

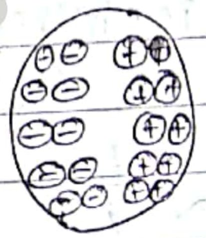
Dielectric is also like an insulator but it behaviour is modified in the electric field. The only difference is that electricity cannot pass through the insulator by any method but electricity can pass through dielectric.

Types of Dielectric:-

[1] polar dielectric:-

↳ The dielectric substances in which centre of gravity of positive charge do not coincide with C.G. of Negative charge is called polar dielectric.

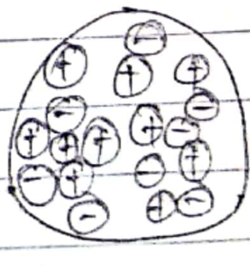
For example:- H_2O, HCl, CO_2 are polar dielectric.



[2] Non-polar dielectric:-

↳ The dielectric substances in which centre of gravity of positive charge coincide with C.G. of Negative charge is called non-polar dielectric.

For example:- O_2, N_2, etc.



Action of Dielectric on a Capacitor:-

→ Suppose a Capacitor with two metallic plates A is +vely and B is negatively Charged as shown is figure. Suppose E_0 be the uniform electric field between the plates of the parallel plate Capacitor.

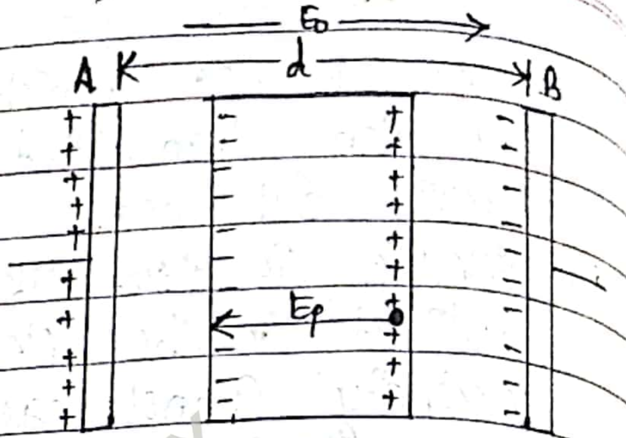


fig:- Action of Dielectric

We now introduce a dielectric slab between the plates of the Capacitor. Due to electrostatic induction, opposite charges are produced at the two opposite surfaces of the slab. The induced charge produces a polarized electric field E_p, which is opposite to E_0.

Thus, the resultant electric field between the plates becomes, $E = E_0 - E_p$

i.e, the electric field between the two plates decreases when the dielectric is introduced. As a result, the p.d. between the plates of the Capacitor reduces. [$E = V/d$]

Since, $C \propto \frac{1}{V}$

Therefore, the Capacitance of the Capacitor increases when the dielectric is introduced between the plates of Capacitor. This is the action of dielectric on the Capacitor.