CHAPTER 18

DISPERSION :-

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Dispersion of light:-

↳ The phenomenon of splitting of white light into it's Constituent Colours is called dispersion of light. It can be remembered by the word VIBGYOR

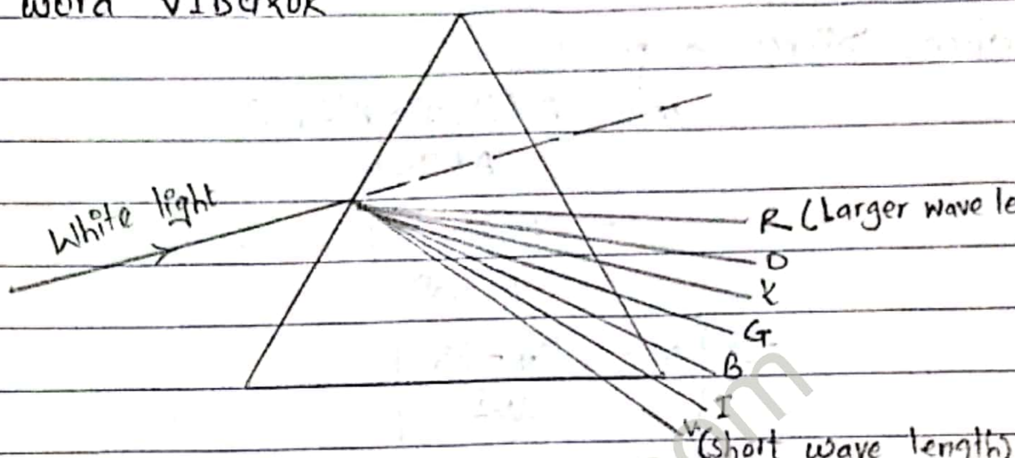

Fig:- Dispersion of light

Angular Dispersion:-

↳ The angular dispersion for two different Colours of light is defined as the difference between angle of deviation for them.

For example, Angular dispersion between red light and violet light through a prism. i.e, $\delta_v - \delta_r$

For Small angled prism,

$$\delta_v = A(\mu_v - 1)$$

$$\delta_r = A(\mu_r - 1)$$

$$\therefore \delta_v - \delta_r = A(\mu_v - 1) - A(\mu_r - 1)$$

$$\Rightarrow \delta_v - \delta_r = A(\mu_v - \mu_r)$$

This is angular dispersion between red & violet light through prism.

Dispersive power :-

↳ The dispersive power of prism is defined as the ratio of angular dispersion for red & violet light to the deviation for mean light.

i.e, dispersive power $(\omega) = \dfrac{\delta_v - \delta_r}{\delta}$

Since, For small angled prism,

$$\delta_v = A(\mu_v - 1)$$

$$\delta_r = A(\mu_r - 1)$$

$$\delta = A(\mu - 1)$$

then, Dispersive power,

$$\omega = \frac{A(\mu_v - 1) - A(\mu_r - 1)}{A(\mu - 1)}$$

$$= \frac{A(\mu_v - \mu_r)}{A(\mu - 1)}$$

$$\Rightarrow \boxed{\omega = \frac{\mu_v - \mu_r}{\mu - 1}}$$

Chromatic Aberration in lenses:-

↳ The lens can be supposed to be made up of large number of small angle prism. When white light is incident on it, due to the prism action of the lens it disperses light & different colours are focused at different point. This inability of lens to focus all colours at a single point is called chromatic aberration.

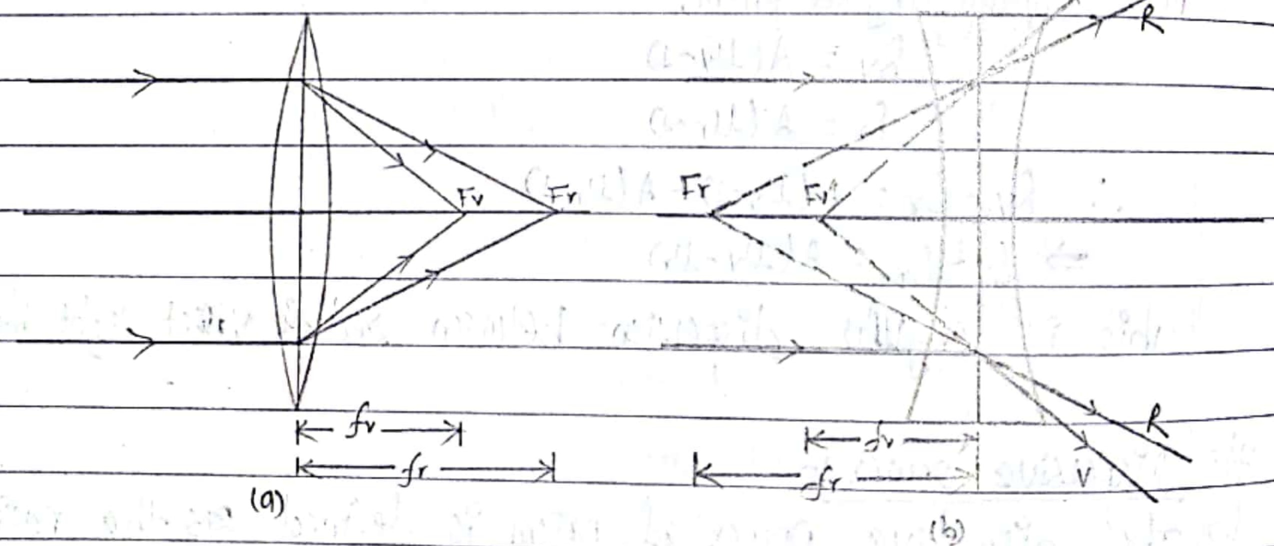

Fig:- Chromatic aberration in (a) Convex lens (b) Concave lens

Let us Consider a white light is incident on a Convex/concave lens parallel to the principal axis, the violet Colour is focused nearer than red Colour and Other Colours are focused in between them as shown in figure. Let, f_v, f_r & f are the focal length for violet, red & mean Colour of light then by lens maker formula,

$$\frac{1}{f_v} = (\mu_v - 1)\left[\frac{1}{R_1} + \frac{1}{R_2}\right] \quad \text{---- (i)}$$

$$\frac{1}{f_r} = (\mu_r - 1)\left[\frac{1}{R_1} + \frac{1}{R_2}\right] \quad \text{---- (ii)}$$

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} + \frac{1}{R_2}\right] \quad \text{---- (iii)}$$

From eq^n(iii)

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{f(\mu - 1)} \quad \text{---- (iv)}$$

Using eq^n(iv) & putting in eq^n(i) & (ii) then, substracting them,

$$\frac{1}{f_v} - \frac{1}{f_r} = \frac{\mu_v - 1}{f(\mu - 1)} - \frac{\mu_r - 1}{f(\mu - 1)}$$

or, $$\frac{f_r - f_v}{f_v f_r} = \frac{\mu_v - 1 - \mu_r + 1}{f(\mu - 1)}$$

or, $$f_r - f_v = \frac{(\mu_v - \mu_r) f_v f_r}{f(\mu - 1)} \quad \text{---- (v)}$$

Also, $\sqrt{f_v f_r} = f$ [Total length of mean light]

$$\Rightarrow f_r - f_v = \frac{\mu_v - \mu_r}{f(\mu - 1)} f^2$$

$$= \left[\frac{\mu_v - \mu_r}{\mu - 1}\right] f$$

$$\Rightarrow \boxed{f_r - f_v = \omega f} \quad \text{---- (vi)} \quad \left[\because \omega = \frac{\mu_v - \mu_r}{\mu - 1}\right]$$

Hence, Chromatic aberration is the product of dispersive power & focal length for mean Colour of light.

Achromatic Combination of lenses:-

↳ The combination of two thin lenses in which their combination is free from chromatic aberration is called the achromatic combination of lenses.

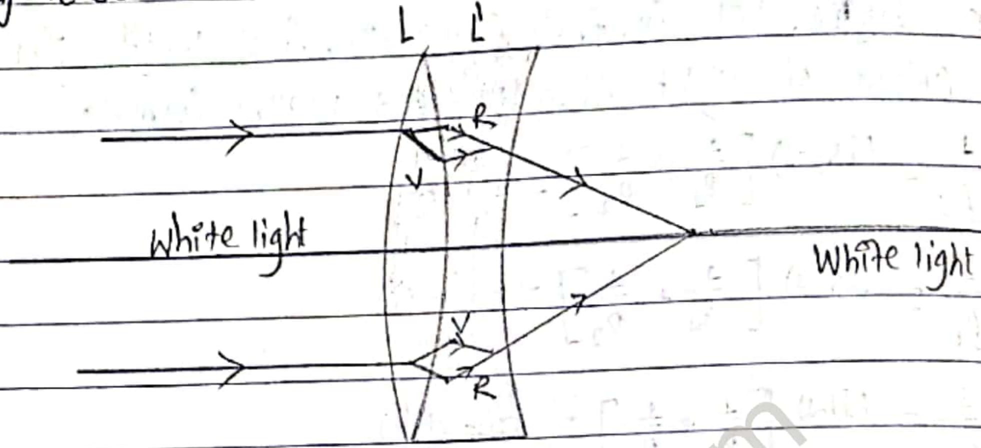

Fig:- Achromatic Combination of lenses

Let us consider two thin lenses 'L' and 'L'' of dispersive power ω & ω' respectively placed in contact with each other as shown in figure. Also, Let μ_v, μ & μ_r are the refractive index of L for violet, mean & red colour respectively. and f_v, f & f_r are the focal lengths of respective colours. Similarly μ_v', μ', μ_r' & f_v', f', f_r' are corresponding quantities of L'. Then,

We have for lens 'L',

$$\frac{1}{f} = (\mu-1)\left[\frac{1}{R_1}+\frac{1}{R_2}\right]$$

or. $$\frac{1}{R_1}+\frac{1}{R_2} = \frac{1}{f(\mu-1)} \quad \text{—— (i)}$$

Similarly, For lens L',

$$\frac{1}{R_1'}+\frac{1}{R_2'} = \frac{1}{f'(\mu'-1)} \quad \text{—— (ii)}$$

Now, focal length for violet colour due to each lenses L & L' are:-

$$\frac{1}{f_v} = (\mu_v-1)\left[\frac{1}{R_1}+\frac{1}{R_2}\right]$$

$$\Rightarrow \frac{1}{f_v} = \frac{\mu_v-1}{f(\mu-1)} \quad \text{—— (iii)} \quad [\text{Using eq}^n \text{(i)}]$$

And, $$\frac{1}{f'_v} = (\mu'_v - 1)\left[\frac{1}{R'_1} + \frac{1}{R'_2}\right]$$

$$\Rightarrow \frac{1}{f'_v} = \frac{\mu'_v - 1}{f'(\mu'-1)} \quad \text{(iv)} \qquad [\text{Using eq}^n \text{(iii)}]$$

Again, Combined focal length for violet colour,

$$\frac{1}{F_v} = \frac{1}{f_v} + \frac{1}{f'_v}$$

$$\frac{1}{F_v} = \frac{\mu_v - 1}{f(\mu-1)} + \frac{\mu'_v - 1}{f'(\mu'-1)} \quad \text{(v)}$$

Similarly, Combined focal length for red colour,

$$\frac{1}{F_r} = \frac{\mu_r - 1}{f(\mu-1)} + \frac{\mu'_r - 1}{f'(\mu'-1)} \quad \text{(vi)}$$

For, Achromatic Combination, we have, $F_v = F_r$

$$\Rightarrow \frac{1}{F_v} = \frac{1}{F_r}$$

$$\Rightarrow \frac{\mu_v - 1}{f(\mu-1)} + \frac{\mu'_v - 1}{f'(\mu'-1)} = \frac{\mu_r - 1}{f(\mu-1)} + \frac{\mu'_r - 1}{f'(\mu'-1)}$$

or, $$\frac{\mu_v - \mu_r}{f(\mu-1)} + \frac{\mu'_v - \mu'_r}{f'(\mu'-1)} = 0$$

or, $$\frac{\omega}{f} + \frac{\omega'}{f'} = 0$$

$$\Rightarrow \boxed{\frac{\omega}{f} = \frac{-\omega'}{f'}} \quad \text{(vii)}$$

Which is the required condition for achromatic combination of two lenses.

$$\Rightarrow \boxed{\frac{\omega}{\omega'} = \frac{-f}{f'}}$$