CHAPTER 17

LENSES:-

↳ A Lense is a piece of transparent refracting material bounded by two surfaces out of which at least one is curved. E.g:- glass lens, diamond lens, etc. Lens is divided into two classes:-

i) Convex lens (or Converging lens)
ii) Concave lens (or diverging lens)

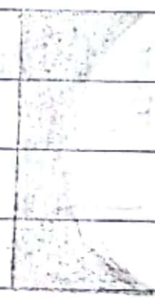
Convex lens:-

↳ If the lens is thick at centre and thin at edges, is called convex lens. It is of three types:-

Bi-convex lens	plano-convex lens	Concavo-convex lens
		

Concave lens:-

↳ If the lens is thin at centre and thick at edges, is called Concave len. It is also of three types:-

Bi-concave lens	plano-concave Lens	Convexo-concave lens
		

Lens Formula:-

↳ The formula which shows the relation between object distance, image distance and focal length of a lens is called lens formula. And is given by,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

★ proof:-

[1] Convex Lens (Real image):-

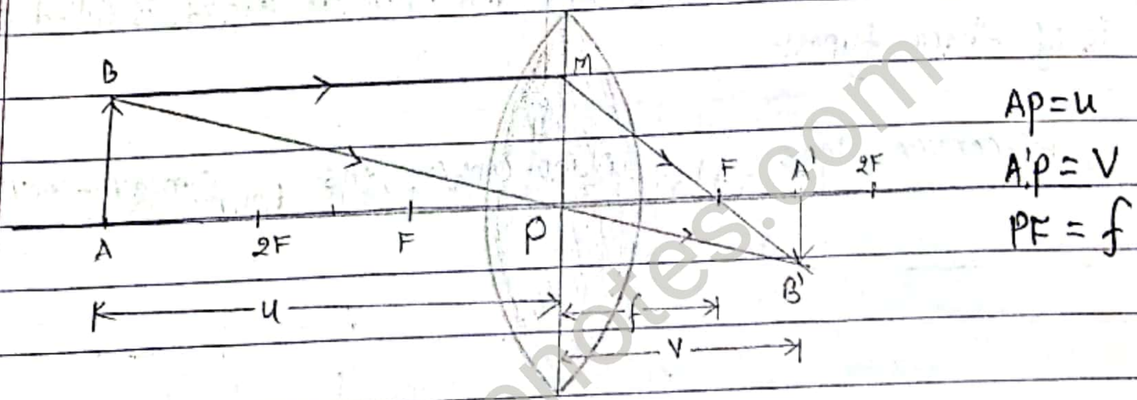


Fig:- Real image formed by a convex lens

Let us consider an object (AB) is placed ⊥ʳ to the principle axis of convex lens of focal length 'f'. Also let, A'B' be the real image formed by that lens. Then,

In above figure, $\Delta A'B'P \sim \Delta ABP$.

So, $\frac{A'B'}{AB} = \frac{A'P}{AP} = \frac{v}{u}$ ——(i)

Also, $\Delta A'B'F \sim \Delta MPF$

So, $\frac{A'B'}{MP} = \frac{A'F}{PF} \Rightarrow \frac{A'B'}{AB} = \frac{A'F}{PF}$ [∵ AB = MP]

Or, $\frac{A'B'}{AB} = \frac{A'P - PF}{PF} = \frac{v-f}{f}$ ——(ii)

From eqⁿ (i) & (ii):

$\Rightarrow \frac{v}{u} = \frac{v-f}{f} \Rightarrow vf = uv - uf$

$\Rightarrow uv = vf + uf$ ——(iii)

Dividing both sides of eqⁿ(iii) with 'uvf'.

Then we get,

or, $\frac{uv}{uvf} = \frac{vf}{uvf} + \frac{uf}{uvf}$

$\Rightarrow \boxed{\frac{1}{f} = \frac{1}{u} + \frac{1}{v}}$ ——(iv)

Which is required lens formula.

[2] Convex Lens (Virtual image):-

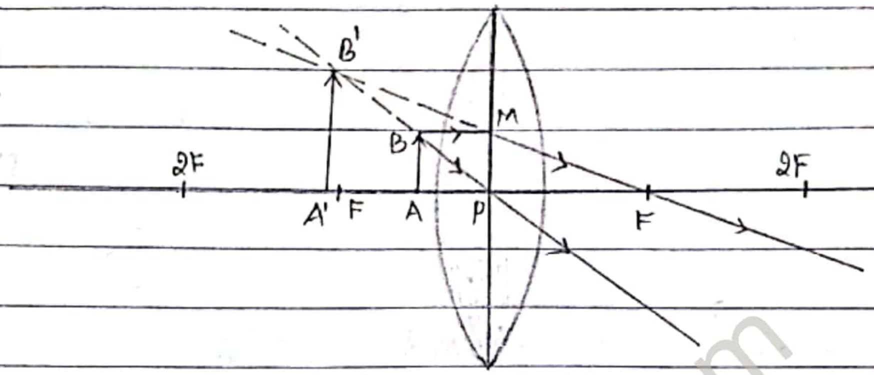


Fig:- Virtual image formed by Convex lens

Let us consider an object AB is placed perpendicularly to the principal axis of Convex Lens of focal length 'f'. Also, Let, A'B' be the virtual image formed by that lens. Then,

In above figure, $\Delta A'B'P \sim \Delta ABP$

So, $\frac{A'B'}{AB} = \frac{A'P}{AP} = \frac{-V}{u}$ —— (i)

Also,

$\Delta A'B'F \sim \Delta MPF$

So, $\frac{A'B'}{MP} = \frac{A'F}{PF}$

or, $\frac{A'B'}{AB} = \frac{A'F}{PF}$ [$\because$ MP = AB]

or, $\frac{A'B'}{AB} = \frac{A'P + PF}{PF}$

$\Rightarrow \frac{A'B'}{AB} = \frac{-V+f}{f}$ —— (ii)

From eq^n (i) & (ii) we get,

$\Rightarrow \frac{-V}{u} = \frac{-V+f}{f}$

or, $-Vf = -uV + uf$

or, $uV = Vf + uf$ —— (iii)

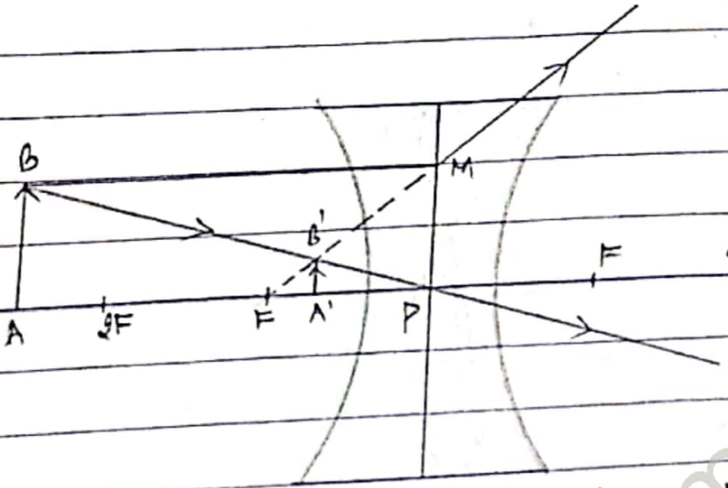
Dividing both sides by 'uvf' of eq^n (iii)

$\Rightarrow \frac{uV}{uVf} = \frac{Vf}{uVf} + \frac{uf}{uVf}$

$\Rightarrow \boxed{\frac{1}{f} = \frac{1}{u} + \frac{1}{V}}$ —— (iv)

Which is required lens formula.

[3] Concave Lens (Virtual image):-



$AP = u$
$A'P = -v$
$PF = -f$

Fig:- Virtual image formed by Concave lens

Let us consider, an object AB is placed perpendicularly to the principal axis of Concave lens of focal length 'f'. Also, let. A'B' be the virtual image formed by that lens. Then,

In above figure, $\Delta A'B'P \sim \Delta ABP$

So, $\dfrac{A'B'}{AB} = \dfrac{A'P}{AP} = \dfrac{-v}{u}$ —— (i)

Also,

$\Delta A'B'F \sim \Delta MPF$

So, $\dfrac{A'B'}{MP} = \dfrac{A'F}{PF}$

or. $\dfrac{A'B'}{AB} = \dfrac{A'F}{PF}$ [$\because$ $MP = AB$]

or, $\dfrac{A'B'}{AB} = \dfrac{PF - A'P}{PF}$

$\Rightarrow \dfrac{A'B'}{AB} = \dfrac{-f + v}{-f}$ —— (ii)

From eq^n (i) & (ii); we get,

$\Rightarrow \dfrac{-v}{u} = \dfrac{-f+v}{-f}$

or, $vf = -uf + uv$

or, $uv = vf + uf$ —— (iii)

Dividing both sides of eq^n (iii) by uvf.

Then,

or, $\dfrac{uv}{uvf} = \dfrac{vf}{uvf} + \dfrac{uf}{uvf}$

$\Rightarrow \boxed{\dfrac{1}{f} = \dfrac{1}{u} + \dfrac{1}{v}}$ —— (iv)

Which is required lens formula.

Linear Magnification:-

↳ Linear magnification is defined as the ratio of the size of the image to the size of object.

OR,

↳ Linear magnification is also defined as the ratio of image distance to the object distance. It is denoted by 'm'.

i.e,
$$m = \frac{I}{O} = \frac{v}{u}$$

Where

I = Image height(size), v = Image distance
O = Object height (size) & u = Object distance

If 'N' numbers of lenses are combined coaxially, the combined magnification is written as,

$$m = m_1 \times m_2 \times - - - \times m_N$$

Where, $m_1, m_2, ---, m_N$ be the magnification produced by N numbers of lenses

Power of lens:-

↳ The reciprocal of focal length of a lens is called power of lens. If focal length = f, expressed in metre, then the power of lens denoted by P is expressed as,

$$P = \frac{1}{f(\text{metre})}$$

The SI unit of power is m^{-1} which is called dioptre(D).

When, $f = 1$ metre, then, $P = \frac{1}{1m} = 1\ D$

⇒ So, If the focal length of lens is one metre then the power of a lens is called one dioptre.

Len's Maker's Formula:-

↳ The relation which shows the relation between focal length of lens radii of curvature of two surfaces of lens and the refractive index of the material of lens is called Lens Maker's formula.

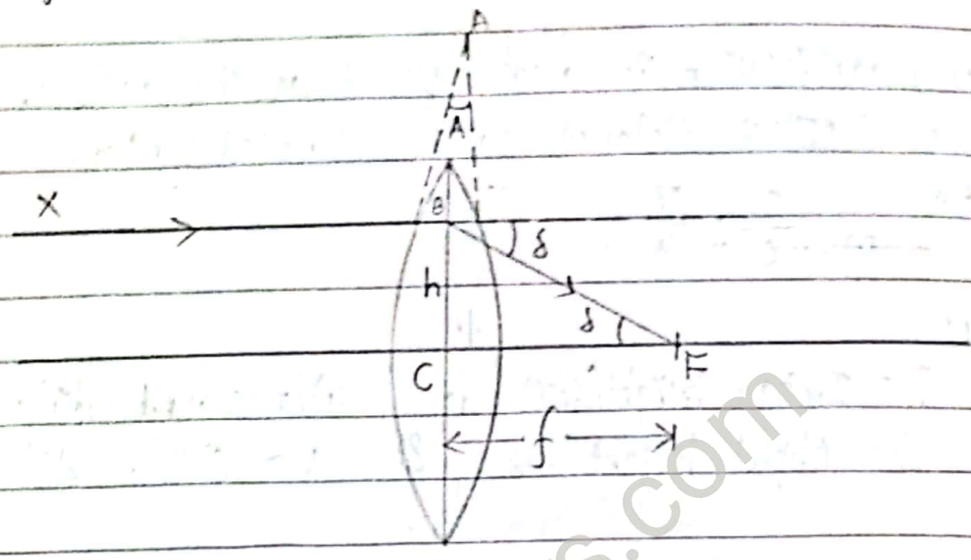


Fig:- Refraction through lens.

Let us consider, a thin convex lens of focal length 'f'. A ray of light XB parallel to the principal axis strikes the point 'B' at height 'h' above the opticle centre 'c' as shown in figure. This ray after refraction through lens meets principal axis at Focus 'F' by making angle of deviation 'δ'.

From figure,

$$\tan\delta = \frac{h}{f}$$

For small δ, $\tan\delta \approx \delta$

$$\Rightarrow \delta = h/f \quad \text{—— (i)}$$

Also, lens is supposed to be small angle prism. Then.

$$\delta = A(\mu-1) \quad \text{—— (ii)}$$

Now, From eqn (i) & (ii);

$$\frac{h}{f} = A(\mu-1)$$

$$\Rightarrow \frac{1}{f} = (\mu-1)\frac{A}{h} \quad \text{—— (iii)}$$

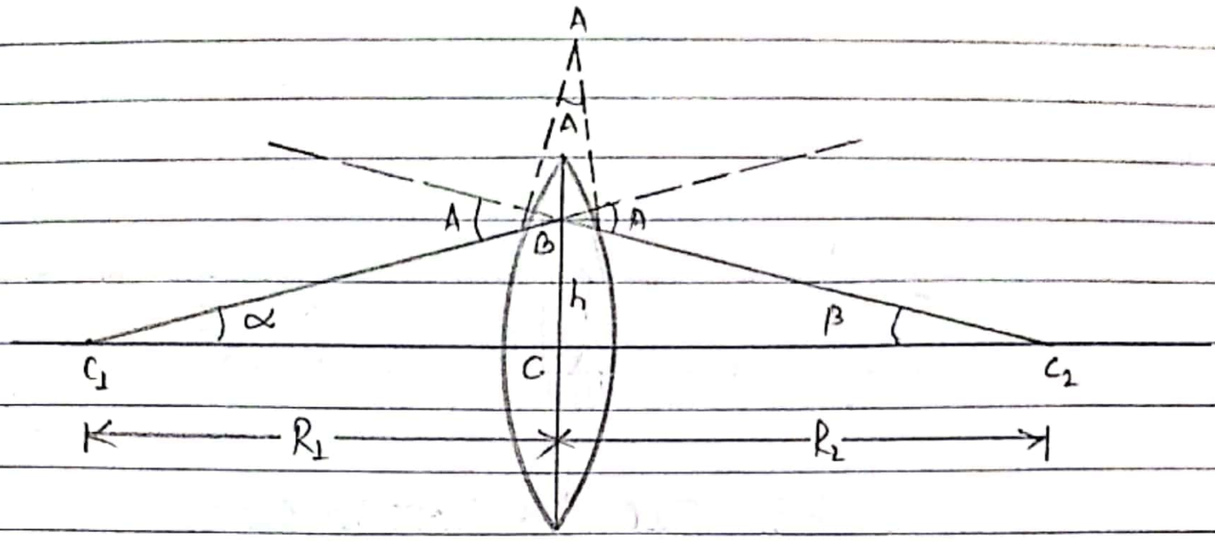


Fig:- Lens as a small angled prism

Let us consider, two light rays passing through C_1 & C_2 (where C_1 & C_2 are centre of curvature) meets at point B of the lens. Also let, α & β are the angle made by BC_1 and BC_2 with principal axis respectively.

From figure, $CC_1 = R_1$ & $CC_2 = R_2$

& $A = \alpha + \beta$ ——— (iv)

Also, In Right angle ΔBCC_1,

$$\tan\alpha = \frac{h}{R_1}$$

For small α, $\tan\alpha \approx \alpha$

$$\Rightarrow \alpha = \frac{h}{R_1} \text{ ——— (v)}$$

Again, In Right angle ΔBCC_2,

$$\tan\beta = \frac{h}{R_2}$$

For small β, $\tan\beta \approx \beta$

$$\Rightarrow \beta = \frac{h}{R_2} \text{ ——— (vi)}$$

Using eq^n (v) & (vi) in eq^n (iv)

$$\Rightarrow A = \frac{h}{R_1} + \frac{h}{R_2}$$

$$\Rightarrow \frac{A}{h} = \frac{1}{R_1} + \frac{1}{R_2} \text{ ----- (vii)}$$

Again, Using eq^n (vii) in eq^n (iii);

$$\Rightarrow \frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} + \frac{1}{R_2}\right] \text{ ——— (viii)}$$

Which is known as lens Maker's formula or equation.

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Focal length of Combined Lens :-

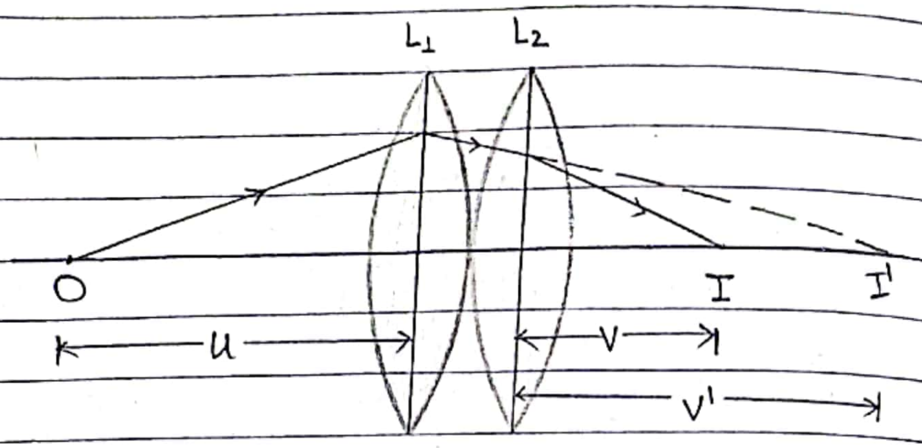

Fig:- Two thin lenses in contact

Let us consider, two thin lenses L_1 & L_2 with their respective focal length f_1 & f_2 are placed in contact with each other as shown in figure. Also consider a point object 'O' placed on the principal axis of the lens system. The lens L_1 form a real image of object 'O' then, using lens formula for lens 'L_1',

$$\frac{1}{f_1} = \frac{1}{u} + \frac{1}{v'} \quad \text{—— (i)}$$

The image 'I' found by lens 'L_1' act as virtual object for lens 'L_2' and which form final image at point I. Then, using lens formula for lens 'L_2',

$$\frac{1}{f_2} = \frac{1}{-v'} + \frac{1}{v} \quad \text{—— (ii)}$$

Adding eq^n (i) & (ii);

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{u} + \frac{1}{v'} - \frac{1}{v'} + \frac{1}{v}$$

$$\Rightarrow \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{u} + \frac{1}{v} \quad \text{—— (iii)}$$

If 'F' be the combined focal length for both lenses L_1 & L_2;

$$\Rightarrow \frac{1}{F} = \frac{1}{u} + \frac{1}{v} \quad \text{—— (IV)}$$

From eq^n (iii) & (iv)

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

For 'N' lenses;

$$\Rightarrow \boxed{\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \cdots + \frac{1}{f_N}} \quad \text{—— (V)}$$

$$\Rightarrow \boxed{P = P_1 + P_2 + \cdots + P_N} \quad \text{—— (vi)}$$

Hence, eq^n (v) helps to determine equivalent focal length & eq^n (vi) helps to find power of combined lenses.