CHAPTER 10 Thermal Expansion :-

↳ When a body get heat, it's amplitude of vibration increases. So that the length, area and volume increases.

When Length increase → Called linear expansion.
When area increase → called Superficial expansion
When Volume increase → Called Cubical expansion

[1] Co-efficient of Linear expansion (Linear expansivity) (α) :-

↳ Let us consider a rod of initial length 'l_1' at temperature 'θ_1'. When its temperature increase to 'θ_2', then its length becomes 'l_2'.

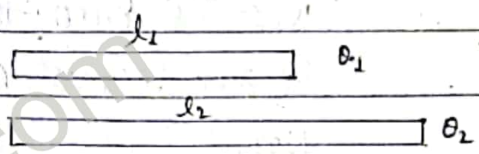

fig: Linear expansion of rod.

Now,
Change in Length of rod, $\Delta l = l_2 - l_1$
change in temperature, $\Delta\theta = \theta_2 - \theta_1$

Experimently, It has been found that;

[I] Change in Length of rod is directly proportional to it's original length.
i.e., $\Delta l \propto l_1$ ---- (i)

[II] Change in length of rod is directly proportional to change in temperature.
i.e., $\Delta l \propto \Delta\theta$ ----- (ii)

Combining eqⁿ (i) and eqⁿ (ii), we get,
$$\Delta l = \alpha\, l_1 \Delta\theta \quad \text{----- (iii)}$$

Where, α is proportionality constant called co-efficient of linear expansion.

From eqⁿ (iii),
$$\alpha = \frac{\Delta l}{l_1 \Delta\theta}$$

$\Rightarrow \boxed{\alpha = \Delta l}$ [$\because$ If $l_1 = 1m$ & $\Delta\theta = 1°C$ or $1K$]

Thus, co-efficient of linear expansion of the material of the rod is defined as the change in length per unit original length per unit change in temperature. And it is numerically equal to change in

of a unit rod when its temperature change by $1^\circ C$ or $1K$.
Length

Also, from eq^n (III)

$$l_2 - l_1 = \alpha\, l_1 (\theta_2 - \theta_1)$$

$$\Rightarrow l_2 = l_1 \{1 + \alpha(\theta_2 - \theta_1)\} \text{ ---- (IV)}$$

[2] Co-efficient of Superficial expansion (Superficial expansivity) (β):-

↳ Let us consider a solid square of initial length 'l_1' & area 'A_1' at temperature 'θ_1'. When its temperature increase to 'θ_2'. Then its area becomes 'A_2' of length 'l_2'.

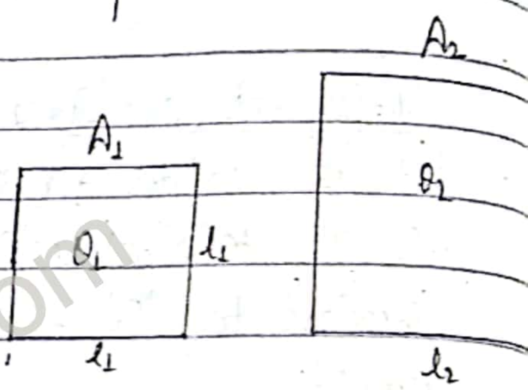

fig: Superficial expansion of Solid Square

Now,

change in area of Square, $\Delta A = A_2 - A_1$

& change in temperature, $\Delta\theta = \theta_2 - \theta_1$

Experimently, It has been found that,

[I] Change in area of square is directly proportional to its original area.

i.e., $\Delta A \propto A_1$ ---- (i)

[II] Change in area of Square is directly proportional to Change in temperature.

i.e., $\Delta A \propto \Delta\theta$ ----- (ii)

Combining eq^n (i) & (ii); we get,

$$\Delta A \propto A_1 \Delta\theta$$

$$\Rightarrow \boxed{\Delta A = \beta A_1 \Delta\theta} \text{ ---- (iii)}$$

Where, β in proportinality constant called Co-efficient of Superficial expansion.

Also, from eq^n (iii)

$$\beta = \frac{\Delta A}{A_1 \Delta\theta}$$

$\Rightarrow \boxed{\beta = \Delta A}$ [$\because$ If $A_1 = 1\, m^2$ & $\Delta\theta = 1^\circ C$ or $1K$]

Thus, co-efficient of Superficial expansion of solid square is defined as change in area per unit initial area per unit change in temperature. And numerically, it is equal to change in Area of a unit Square when its temperature change by $1^\circ C$ or $1K$

Also, from eqⁿ (iii);

$$\mathbf{A_2 - A_1} = \cancel{\beta} A_1(\theta_2 - \theta_1)$$

$$\Rightarrow A_2 = A_1\{1 + \beta(\theta_2 - \theta_1)\} \text{ ---- } \boxed{IV}$$

[3] Co-efficient of Cubical expansion (γ) (Cubical expansivity):-

↳ Let us Consider a solid Cube of initial Volume 'V_1' & length 'l_1' at temperature 'θ_1'. When its temperature increases to 'θ_2' then its Volume becomes 'V_2' of length 'l_2' then, ~~it tem~~

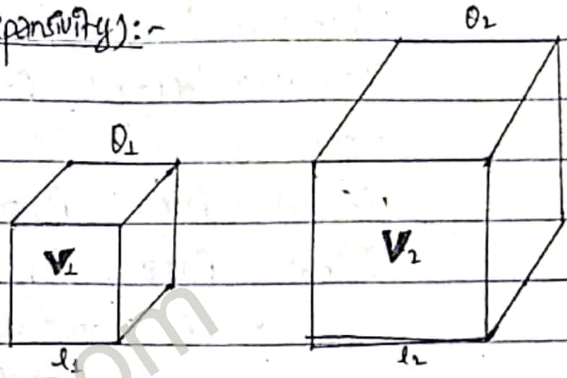

Fig: Cubical expansion of Solid Cube

Change in Volume of Cube, $\Delta V = V_2 - V_1$

Change in temperature, $\Delta\theta = \theta_2 - \theta_1$

Experimetly it has been found that,

(I) Change in Volume is directly proportional to it original volume, i.e, $\Delta V \propto V_1$ ---- (i)

(II) Change in Volume is directly proportional to change in temperature, i.e, $\Delta V \propto \Delta\theta$ --- (ii)

Combining eqⁿ (i) & (ii), $\Delta V \propto V_1 \Delta\theta$

$$\Rightarrow \boxed{\Delta V = \gamma V_1 \Delta\theta} \text{ ---- (iii)}$$

Where, γ is proportionality Constat Called co-efficient of cubical expansion.

Also, from eqⁿ (iii);

$$\gamma = \frac{\Delta V}{V_1 \Delta\theta}$$

[If $V_1 = 1\,m^3$ and $\Delta\theta = 1°c$ or $1K$]

then,

$$\boxed{\gamma = \Delta V}$$

Thus, co-efficient of Cubical expansion of Solid Cube is defined as change in Volume per unit original Volume per unit change in temperature. And, numerically, it is equal to change in volume of unit when its temperature change by 1°c or 1K.

Also, from eqⁿ (iii)

$$V_2 - V_1 = \gamma V_1(\theta_2 - \theta_1)$$

$$\Rightarrow \boxed{V_2 = V_1[1 + \gamma\{\theta_2 - \theta_1\}]} \text{ ---- } \boxed{IV}$$

Relation between α, β & γ.

[1] Relation between α & β :-

↳ Let us consider a solid square of initial length 'l_1' & area 'A_1' at temperature 'θ_1'. When its temperature increase to 'θ_2', then, its area becomes 'A_2' of length 'l_2'.

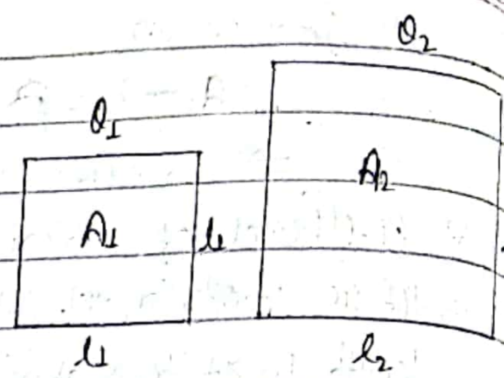

fig: Superficial expansion.

Now,

Initial Area, $A_1 = l_1^2$ ---- (i)

Final Area, $A_2 = l_2^2$ ---- (ii)

From Superficial expansivity;

$$A_2 = A_1 \{1 + \beta \Delta\theta\} \text{ ---- (iii)}$$

Also, from linear expansivity;

$$l_2 = l_1 \{1 + \alpha \Delta\theta\} \text{ ---- (iv)}$$

Using eq^n (iv) in eq^n (ii)

$$A_2 = [l_1 \{1 + \alpha\Delta\theta\}]^2$$

$$= l_1^2 \{1 + 2\alpha\Delta\theta + \alpha^2 \Delta\theta^2\}$$

Since, α is small quantity, so its higher term can be neglected,

$$A_2 = l_1^2 \{1 + 2\alpha\Delta\theta\}$$

$$A_2 = A_1 \{1 + 2\alpha\Delta\theta\} \text{ ---- (v)} \quad [\because \text{Using eq}^n \text{ (i)}]$$

Comparing eq^n (iii) and eq^n (v), we get;

$$\Rightarrow \boxed{\beta = 2\alpha}$$

This is required relation between α & β.

[2] Relation between α & γ :-

↳ Let us consider a solid cube of initial volume 'V_1' & length 'l_1' at temperature 'θ_1'. When its temperature increases to 'θ_2', then its volume becomes 'V_2' of length 'l_2'.

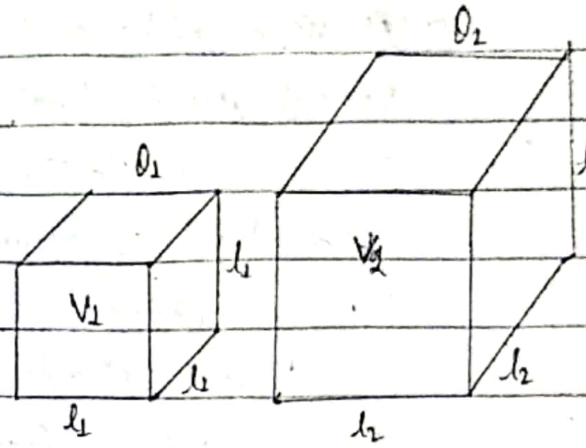

fig: Cubical expansion

Now,

Initial Volume, $V_1 = l_1^3$ ----(i)

& Final Volume, $V_2 = l_2^3$ ----(ii)

From cubical expansivity;

$$V_2 = V_1\{1+\gamma\Delta\theta\} \text{ ----(iii)}$$

Also, from linear expansivity

$$l_2 = l_1\{1+\alpha\Delta\theta\} \text{ ----(iv)}$$

Using eq^n (iv) in eq^n (ii)

$$V_2 = [l_1\{1+\alpha\Delta\theta\}]^3$$
$$= l_1^3\{1+3\alpha\Delta\theta+3\alpha^2\Delta\theta^2+\alpha^3\Delta\theta^3\}$$

Since, α is small quantity. So, its higher powers term can be neglected.

$$V_2 = l_1^3\{1+3\alpha\Delta\theta\}$$
$$V_2 = V_1\{1+3\alpha\Delta\theta\} \text{ ----(v)} \quad [\because \text{Using eq}^n \text{ (i)}]$$

Comparing eq^n (iii) & (v), we get,

$$\boxed{\gamma = 3\alpha}$$

This is required relation between α and γ.

We have,

$$\beta = 2\alpha$$
$$\therefore \alpha = \beta/2 \text{ ----(I)}$$

Also, we have,

$$\gamma = 3\alpha$$
$$\therefore \alpha = \frac{\gamma}{3} \text{ ----(II)}$$

From eq^n (I) & (II)

$$\Rightarrow \boxed{\alpha = \frac{\beta}{2} = \frac{\gamma}{3}}$$

This is required relation between α, β & γ.

Expansion of liquid:-

↳ Let us Consider a liquid kept in a vessel up to Level 'A'. When the vessel is heated initially, the vessel expands and level of liquid falls to 'B'. When heating is continue the liquid, the liquid expands up to level 'C'. Here the expansion from initial level 'A' to final Level 'C' is called apparent expansion. Expansion from B to C is called real expansion. And the expansion from A to B is called vessel expansion.

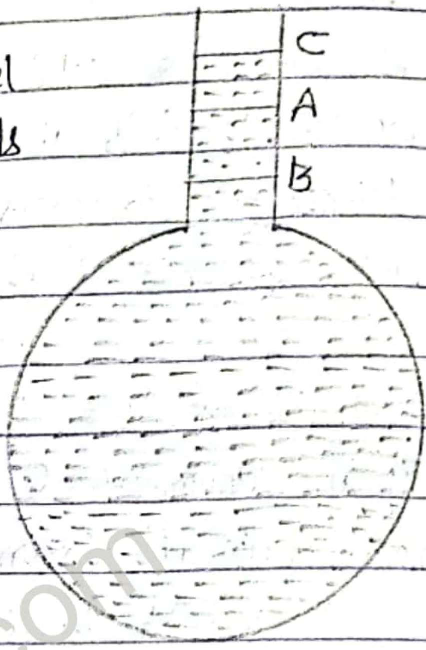

fig: Expansion of Liquid

From figure, $BC = AC + AB$

i.e., Real expansion = Apparent expansion + Vessel expansion

Co-efficient of Real expansion [Real expansivity] (γ_r):-

↳ It is defined as the, "Real change in volume of liquid per unit original volume per unit change in temperature." It is also called absolute expansivity. If ΔV_r be the real change in volume of liquid with initial volume 'V' for change in temperature '$\Delta\theta$'. then, Real expansivity is,

$$\gamma_r = \frac{\Delta V_r}{V \Delta\theta}$$

$$\Rightarrow \boxed{\Delta V_r = \gamma_r V \Delta\theta} \quad \text{------ (i)}$$

Co-efficient of Apparent expansion [Apparent expansivity] (γ_a):-

↳ It is defined as the, "apparent change in volume of liquid per unit original volume per unit change in temperature."

If 'ΔV_a' be the Apparent change in Volume of liquid with initial volume 'V' for change in temperature '$\Delta\theta$'. Then, Apparent expansivity;

$$\gamma_a = \frac{\Delta V_a}{V\Delta\theta}$$

$$\Rightarrow \boxed{\Delta V_a = \gamma_a V \Delta\theta} \text{ ----- (ii)}$$

Co-efficient of Cubical expansion of vessel [expansivity of vessel] (γ_v):-

↳ It is defined as the "change in volume of vessel per unit original Volume per unit change in temperature".

If 'ΔV_v' be the change in volume of vessel with initial Volume 'V' for change in temperature '$\Delta\theta$'. The expansivity of vessel;

$$\gamma_v = \frac{\Delta V_v}{V\Delta\theta}$$

$$\Rightarrow \boxed{\Delta V_v = \gamma_v V\Delta\theta} \text{ ----- (iii)}$$

Since,

$$\Delta V_r = \Delta V_a + \Delta V_v$$

or, $\gamma_r V\Delta\theta = \gamma_a V\Delta\theta + \gamma_v V\Delta\theta$

or, $\gamma_r = \gamma_a + \gamma_v$

$$\Rightarrow \boxed{\gamma_r = \gamma_a + 3\alpha_v}$$

This is required relation between Real expansivity and Apparent expansivity.

Variation of density with temperature:-

↳ Let us Consider a body having mass 'M' and Volume 'V_1' at temperature 'θ_1'. Then, density is given by;

$$\rho_1 = \frac{M}{V_1} \text{ ----- (i)}$$

Suppose the body is heated upto temperature 'θ_2' and its Volume becomes 'V_2' but it's mass remains same. So, density be,

$$\rho_2 = \frac{M}{V_2} \quad \text{----- (ii)}$$

Dividing eqⁿ (ii) by (i)

or, $$\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} \quad \text{----- (iii)}$$

If 'γ' be the Cubical expansivity of the material/body then,

$$V_2 = V_1 \{1 + \gamma \Delta\theta\} \quad \text{--- (iv)}$$

Where, $\Delta\theta = \theta_2 - \theta_1$

Using eqⁿ (iv) in eqⁿ (iii)

or, $$\frac{\rho_2}{\rho_1} = \frac{V_1}{V_1 \{1 + \gamma \Delta\theta\}}$$

$$\Rightarrow \boxed{\rho_2 = \frac{\rho_1}{1 + \gamma\Delta\theta}} \Rightarrow \text{for numerical}$$

or, $$\rho_2 = \rho_1 (1 + \gamma\Delta\theta)^{-1}$$

Using Binomial Expansion and Neglecting higher power term;

$$\boxed{\rho_2 = \rho_1 (1 - \gamma\Delta\theta)} \quad \text{---- (VI)}$$

eqⁿ (VI) gives Variation of density with temperature.

Dulong and Petit's method:-

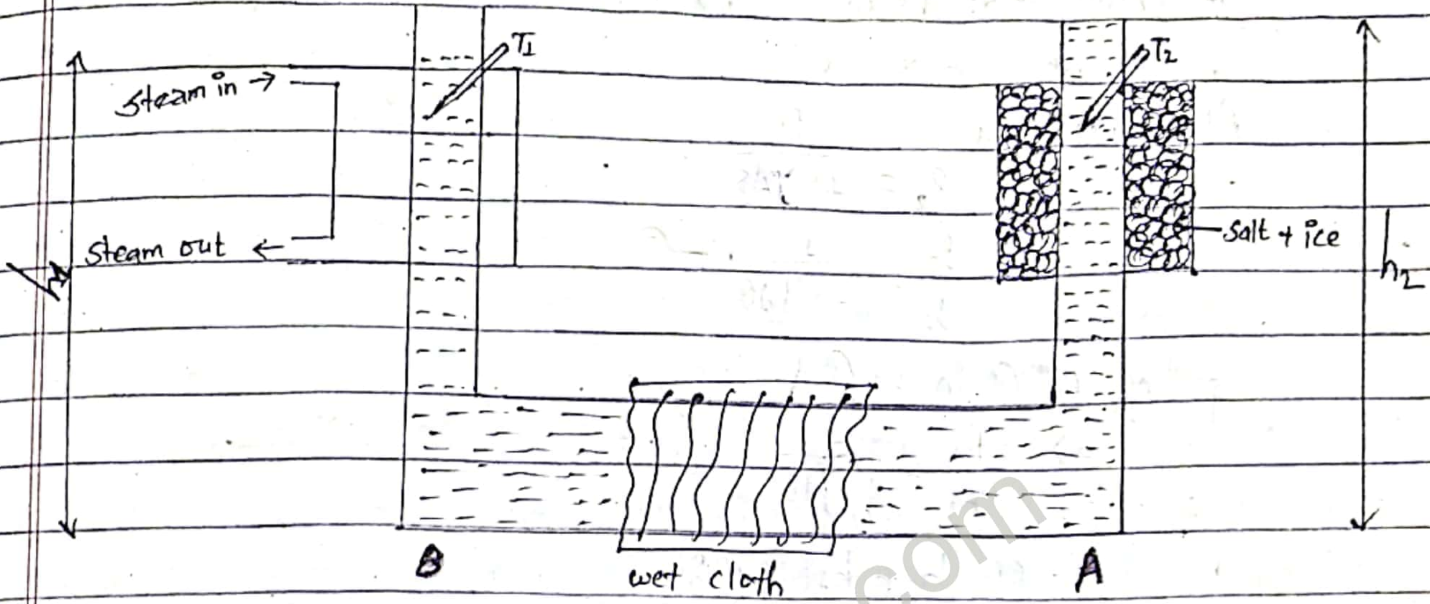

fig: Dulong & petit's method for measuring Real expansivity.

↳ The experiment of Dulong & petit's method depends upon the principle of hydrostatic. It states that, "The height of liquid Column that produce same pressure are inversely proportional to their density.

↳ Let us Consider a U-shaped glass tube filled with liquid. Its one side is kept inside the ice and salt. and other side is kept inside the Steam in and Steam out. The horizontal portion of tube is wrapped with wet cloth to prevent exchange of heat with Surrounding. Two thermometer are provided to note the temperature of A & B side of the tube as shown in figure above.

Let 'h_1', ρ_1, T_1 and h_2, ρ_2, T_2 be the height, density and temperature of liquid at B & A side of glass tube respectively.

Now, From principle of hydrostatic,

pressure of A = pressure of B

$$\Rightarrow h_2 \rho_2 g = h_1 \rho_1 g$$

$$\Rightarrow \frac{h_1}{h_2} = \frac{\rho_2}{\rho_1} \quad ---(i)$$

It shows that height is inversely proportional to density.

i.e., $h \propto \frac{1}{\rho}$

Also, we have,

$$\rho_2 = \frac{\rho_1}{1+\gamma\Delta\theta}$$

$$\frac{\rho_2}{\rho_1} = \frac{1}{1+\gamma\Delta\theta} \quad \text{—(ii)}$$

putting eqn (ii) in eqn (i) . . .

$$\Rightarrow \frac{h_1}{h_2} = \frac{1}{1+\gamma\Delta\theta}$$

or, $h_2 = h_1(1+\gamma\Delta\theta)$

or, $\gamma\Delta\theta = \frac{h_2}{h_1} - 1 = \frac{h_2 - h_1}{h_1}$

$\therefore$ $\boxed{\gamma = \frac{h_2 - h_1}{h_1\,\Delta\theta}}$ —(iii) Using this eqn (iii), we can calculate the real expansivity of liquid.

Numericals :-

Q.1 [A] A glass flask of volume 400 cm^3 is just filled with mercury at 0°C. How much mercury will overflow when the temperature of the system rises to 80°C.

☆ Soln:-

$V_{2m} = ?$, $V_{1m} = V_{1f} = 400\ cm^3$, $\gamma_m = 1.8\times10^{-4}\ K^{-1}$, $\gamma_f = 5.4\times10^{-6}\ K^{-1}$, $V_{2f} = ?$, $\Delta\theta = 80°C$

Now,

$V_{2m} = V_{1m}[1+\gamma_m\Delta\theta]$

$= 400[1+(1.8\times10^{-4}\times80)]$

$= 405.76\ cm^3$

$V_{2f} = V_{1f}[1+\gamma_m\Delta\theta]$

$= 400[1+(5.4\times10^{-6}\times80)]$

$= 400.1728\ cm^3$

Now,

Volume of overflows mercury,

$= V_{2m} - V_{2f}$

$= 405.76\ cm^3 - 400.1728\ cm^3$

$= 5.5872\ cm^3$

#

Q.1[B] A glass vessel of Volume 50 cm^3 is filled with mercury and is heated from 20°C to 60°C. What Volume of mercury will overflow?

☆ Sol^n:-

$V_{2m} = V_{1m}[1+\gamma_m \Delta\theta]$
$= 50[1+(1.8\times10^{-4}\times40)]$
$= 50.36\ cm^3$

$V_{2V} = V_{1V}[1+\gamma_V\Delta\theta]$
$= 50[1+5.4\times10^{-6}\times40)]$
$= 50.0108\ cm^3$

Vol^m of overflow mercury,
$= V_{2m} - V_{2V}$
$= 0.3492\ cm^3$ #

Q.1[C] A glass flask with volume 200 cm^3 is filled to the brim with mercury at 20°C. How much mercury overflows when the temperature of the system is raised to 100°C?

☆ Sol^n:-

$V_{2m} = V_{1m}[1+\gamma_m \Delta\theta]$
$= 200[1+(1.8\times10^{-4}\times80)]$
$= 202.88\ cm^3$

$V_{2f} = V_{1f}[1+\gamma_f\Delta\theta]$
$= 200[1+(5.4\times10^{-6}\times80)]$
$= 200.0864\ cm^3$

Vol^m of mercury overflows,
$= V_{2m} - V_{2f}$
$= 2.7936\ cm^3$

Q.1[D] A Copper vessel with a Volume of exactly 100 m^3 at a temperature of 15°C is filled with glycerin. If the temperature rises to 25°C, how much glycerin will spill out?

☆ Sol^n:-

$V_{2g} = V_{1g}[1+\gamma_g\Delta\theta]$
$= 100[1+(49\times10^{-5}\times10)]$
$= 100.49\ m^3$

$V_{2V} = V_{1V}[1+\gamma_V\Delta\theta]$
$= 100[1+(5.1\times10^{-5}\times10)]$
$= 100.051\ m^3$

Volume of spill out glycerin,
$= V_{2g} - V_{2V}$
$= 0.439\ m^3$ #

Q.1[E] A glass flask of Volume 500 cm^3 is just filled with mercury at 0°C. How much mercury overflows when the temperature of the system is raised to 80°C?

☆☆

$V_{2m} = V_{1m}[1+\gamma_m\Delta\theta]$
$= 500[1+(1.8\times10^{-4}\times80)]$
$= 507.2\ m^3$

$V_{2f} = V_{1f}[1+\gamma_f\Delta\theta]$
$= 500[1+(5.4\times10^{-6}\times80)]$
$= 500.216\ m^3$

Vol^m of overflows mercury,
$= V_{2m} - V_{2f}$
$= 6.984\ m^3$ #

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Q.2[A] A steel wire having length 8m and diameter 4 mm is fixed between two rigid su[pports]. Calculate increase in tension on a wire when temperature falls by 10°c. Where $Y_w = 2\times10^{11}\ N/m^2$; $\alpha_s = 1.2\times10^{-5}\ K^{-1}$; $d = 4mm = 0.004\ m$.

☆ Solⁿ:- $\Delta\theta = 10°c$

We have,

$$Y = \frac{F}{A}\times\frac{L}{\Delta L} \Rightarrow F = \frac{Y_w A_w \Delta l_w}{L_w} = \frac{2\times10^{11}\times\pi d^2\times\alpha_w \cancel{l_w}\Delta\theta}{4\times\cancel{l_w}} = \frac{10^{11}\pi}{2}\times(0.004)^2\times1.2\times10^{-5}\times10$$

$$\therefore \text{Tension} = \frac{192\pi}{2} = 96\pi = 301.6\ N \ \#$$

Q.2[B] Two ends of a steel wire of length 8m and 2mm radius are fixed to two rigid suppo[rts]. Calculate the increase in tension in the wire when temperature falls by 10°c.

☆ Solⁿ:- $Y_w = 2\times10^{11}$; $l_w = 8m$, $r_w = 2mm = 0.002\ m$, $\Delta\theta = 10°c$, $\alpha_w = 1.2\times10^{-5}K^{-1}$

Now,

$$Y_w = \frac{F}{A_w}\times\frac{L_w}{\Delta L_w} \Rightarrow F = \frac{Y_w A_w \Delta L_w}{L_w} = \frac{2\times10^{11}\times\pi r^2\times\alpha_w \cancel{l_w}\Delta\theta}{\cancel{l_w}} = 2\pi\times10^{11}\times(0.002)^2\times1.2\times10^{-5}\times10 = 96\pi = 301.6N$$

\#

Hence, Required tension is 301.6 N #

Q.3[A] A iron rod of length 100 m at 10°c is used to measure a distance of 2km on a day when the temperature is 40°c. Calculate the error in measuring the distance.

☆ Solⁿ:-

Length of rod at 40°c,

$= 100\{1+(16\times10^{-6}\times30)\}$

$= 100.048\ m$

Now, $100m \xrightarrow{40°c} 100.048m$

$1m \xrightarrow{40°c} 1.00048\ m$

$2km \xrightarrow{40°c} 1.00048\times2000\ m$

$= 2000.96\ m$

Hence, Error in measuring the distance,

$= 2000.96m - 2000m$

$\Rightarrow 0.96\ m$ #

Q.4[A] The marking on an aluminium ruler and a brass ruler are perfectly aligned at 0°C. How far apart will the 20.0 cm marks be on the two rulers at 100°C, if precise alignment of the left hand ends of the rulers is maintained?

Soln:-

$l_{2a} = l_{1a}[1+\alpha_a \Delta\theta]$
$= 20[1+(2.4\times10^{-5}\times100)]$
$= 20.048$ cm

$l_{2b} = l_{1b}[1+\alpha_b \Delta\theta]$
$= 20[1+(2\times10^{-5}\times100)]$
$= 20.04$ cm

Difference in distance,
$= l_{2a} - l_{2b}$
$= 0.008$ cm

Thus, 0.008 cm far will be 20.0 cm marked.

Q.4[B] The length of an iron rod is measures by a brass scale. When both of them are at 10°C, the measured length is 50 cm. What is the length of the rod at 40°C when measured by the brass scale at 40°C? ($\alpha_b = 24\times10^{-6}\,°C^{-1}$, $\alpha_i = 16\times10^{-6}\,°C^{-1}$)

Soln:- $\Delta\theta = 40°C - 10°C = 30°C$

Length of rod at 40°C,
$l_{2i} = l_{1i}[1+\alpha_i \Delta\theta]$
$= 50[1+(16\times10^{-6}\times30)]$
$= 50.024$ cm

Length of scale at 40°C,
$l_{2b} = l_{1b}[1+\alpha_b \Delta\theta]$
$= 50[1+(24\times10^{-6}\times30)]$
$= 50.036$ cm

difference in distance, = 0.012 cm
Hence,
Length of rod at 40°C measured by scale is ((50 − 0.012) cm) = 49.988 cm

Q. 4[C] A brass rod of length 0.40 m and steel rod of length 0.60 m, both are initially at 0°C are heated to 75°C. If the increase in lengths is the same for both the rods. Calculate the linear expansivity of brass. ($\alpha_s = 12\times10^{-6}\,K^{-1}$)

Soln:-

$(\Delta l)_{brass} = (\Delta l)_{steel}$

$\Rightarrow l_b \alpha_b \Delta\theta = l_s \alpha_s \Delta\theta = 0.6\times12\times10^{-6}$

$\alpha_b = \frac{7.2\times10^{-6}}{0.4} = 18\times10^{-6}\ K^{-1}$

Thus, linear expansivity of brass (rod) = $18\times10^{-6}\ K^{-1}$.

Q.4[D] An aluminium rod when measured with a steel scale, both being at 25°c appears to be 1m long. If the scale is correct at 0°c, what will be the length of rod at 0°c? $[\alpha_a = 26\times10^{-6} K^{-1}, \alpha_s = 12\times10^{-6} K^{-1}]$

★ Solⁿ:-

Length of steel scale at 25°c, $\Rightarrow l_{25A} = 1.0003\ m,\ l_{0A} = ?$

$L_{25s} = l_{0s}[1+\alpha_s \Delta\theta]$

$= 1[1+(12\times10^{-6}\times25)]$

$= 1.0003\ m$

or, $l_{25A} = l_{0A}[1+\alpha_A \Delta\theta]$

or, $1.0003 = l_{0A}[1+(26\times10^{-6}\times25)]$

$\Rightarrow l_{0A} = 0.99\ m$

Hence, length of rod at 0°c is accurately 0.99 m #

Q.2[C] A Copper wire of diameter 0.5mm is stretched between two points at 25°c. Calculate the increase in tension in the wire if the temperature falls to 0°c. $(Y_c = 1.2\times10^{11} Nm^{-2}$, $\alpha_c = 18\times10^{-6} K^{-1}]$

★ Solⁿ We have,

$Y = \frac{F}{A}\times\frac{l}{\Delta l} \Rightarrow F = \frac{YA\Delta l}{l} = \frac{1.2\times10^{11}\times(\pi r^2)\times\alpha l\Delta\theta}{l} = 1.2\pi\times10^{11}\times\left(\frac{0.0005}{2}\right)^2\times25\times18\times10^{-6} = 10.6N$

Thus, increase in tension is 10.6 N #

Q.5[A] Using the following data, determine the temperature at which wood will just sink in benzene? [Density of benzene at 0°C $= 9\times10^2\ kg\,m^{-3}$ & $\rho_w^{0°c} = 8.8\times10^2\ kg/m^3$.]

★ Solⁿ

$\rho_{2w} = \rho_{2b}$

or, $\frac{\rho_{1w}}{1+\gamma_w\Delta\theta} = \frac{\rho_{1b}}{1+\gamma_b\Delta\theta}$

or, $\frac{8.8\times10^2}{1+[1.5\times10^{-4}\times(T-0)]} = \frac{9\times10^2}{1+[1.2\times10^{-3}\times(T-0)]}$

or, $8.8[1+(1.2T\times10^{-3})] = 9[1+(1.5T\times10^{-4})]$

~~or, $1+(1.2T\times10^{-3}) = 1.02[1+(1.5T\times10^{-4})]$~~

~~or, $1-1.02+(1.2T\times10^{-3}) = [1.53\times10^{-5}T]$~~

~~or, $-0.02+(1.2\times10^{-3})T = (1.53\times10^{-5})T$~~

~~or, $0.00118T = 0.02$~~

~~$T =$~~

or, $8.8+(10.56\times10^{-3}T) = 9+(13.5\times10^{-4}T)$

or, $10.56\times10^{-3}T = 0.2+(13.5\times10^{-4})T$

or, $0.00921T = 0.2$

$\Rightarrow T = 21.71°C$

$= 294.71\ K$ #

Hence, temp of wood is 21.71°c #

Q.5[B] The density of silver at 0°c is 10310 Kg/m^3 and the coefficient of linear expansion is 0.000019 K^{-1}. Calculate its density at 100°c.

☆ Soln:-

$\rho_0 = 10310\ Kg/m^3$, $\alpha = 0.000019\ K^{-1}$, $\rho_{100} = ?$, $\Delta\theta = 100°c$

Now,
$$\rho_{100} = \frac{\rho_0}{1+\gamma\Delta\theta} = \frac{10310}{1+(3\alpha \times 100)} = \frac{10310}{1+(3\times 0.000019 \times 100)} = \frac{10310}{1.0057} = 10251.56\ Kg/m^3$$ #

Hence, density of silver at 100°c is 10251.56 Kg/m^3 #

Q.6[A] A second pendulum made of brass keeps correct time at 10°c. How many seconds it will loss or gain per day when the temperature of its surrounding rises to 35°c?

☆ Soln:-
$$T_{10} = 2\pi\sqrt{\frac{l_{10}}{g}} \quad \text{—(i)} \qquad T_{35} = 2\pi\sqrt{\frac{l_{35}}{g}} \quad \text{—(ii)}$$

dividing eqn (ii) by eqn (i)
$$\frac{T_{35}}{T_{10}} = \sqrt{\frac{l_{35}}{l_{10}}} = \sqrt{\frac{l_{10}\{1+\alpha_b\Delta\theta\}}{l_{10}}} = \sqrt{1+(2\times10^{-5}\times25)} = 1.000249969$$

Again, Required time $= [1.000249969 - 1]\times 24\times60\times60 = 21.6$ sec ^{loss} #

Q.6[B] A brass pendulum clock keeps correct time at 15°c. How many seconds per day it will lose or gain at 0°c?

☆ Soln:-
$$T_{15} = 2\pi\sqrt{\frac{l_{15}}{g}} \quad \text{—(i)} \qquad T_0 = 2\pi\sqrt{\frac{l_0}{g}} \quad \text{—(ii)}$$

dividing eqn (i) by (ii)
$$\frac{T_{15}}{T_0} = \sqrt{\frac{l_{15}}{l_0}} = \sqrt{\frac{l_0[1+\alpha\Delta\theta]}{l_0}} = \sqrt{1+(2\times10^{-5}\times15)} = 1.000149989$$

Now,

Required time $= [1.000149989 - 1]\times24\times60\times60 = 12.96$ sec gain #

Q.6[C] The pendulum of a clock is made of brass. If the clock keeps correct time 15°C how many seconds per day it will lose at 20°C?

A. Soln

$T_{15} = 2\pi\sqrt{\frac{l_{15}}{g}}$ — (i) $\qquad T_{20} = 2\pi\sqrt{\frac{l_{20}}{g}}$ — (ii)

dividing (ii) by (i);

$\frac{T_{20}}{T_{15}} = \sqrt{\frac{l_{20}}{l_{15}}} = \sqrt{\frac{l_{15}\{1+\alpha\Delta\theta\}}{l_{15}}} = \sqrt{1+\alpha\Delta\theta} = \sqrt{1+(2\times10^{-5}\times5)} = 1.000049999$

Again,

Thus, Required lose time $= [1.000049999-1]24\times60\times60$ sec $= 4.32$ sec #

Q.6[D] A clock which has a brass pendulum beats seconds correctly when the temperature of the room is 30°C. How many seconds it will it gain or lose per day when the temperature of the room falls to 10°C? ($\alpha_b = 0.000018\ K^{-1}$)

A. Soln:-

$T_{30} = 2\pi\sqrt{\frac{l_{30}}{g}}$ — (i) $\qquad T_{10} = 2\pi\sqrt{\frac{l_{10}}{g}}$ — (ii)

dividing eqn (i) by eqn (ii)

or, $\frac{T_{30}}{T_{10}} = \sqrt{\frac{l_{30}}{l_{10}}} = \sqrt{\frac{l_{10}\{1+\alpha\Delta\theta\}}{l_{10}}} = \sqrt{1+\alpha\Delta\theta}$

$= \sqrt{1+(0.000018\times20)}$

$= 1.00017998 4$

Again,

Thus, Required time $= [1.000179984-1]\times24\times60\times60$ sec

$= 15.55$ sec gain #