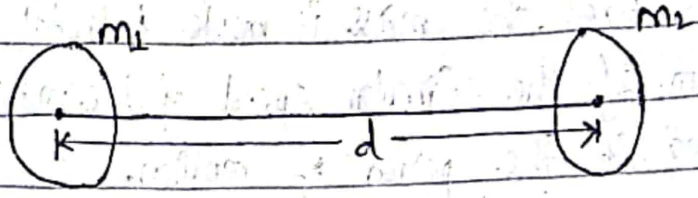
Gravitation:-

Newton's law of Gravitation:-



Let us consider the force of attraction between two bodies of masses m_1 and m_2 separated distance 'd' from their centres is directly proportional to product of their mass and inversely proportional to the square of their separation from their centre

i.e., $F \propto m_1 m_2$ ---- (i)

$F \propto \frac{1}{d^2}$ ---- (ii)

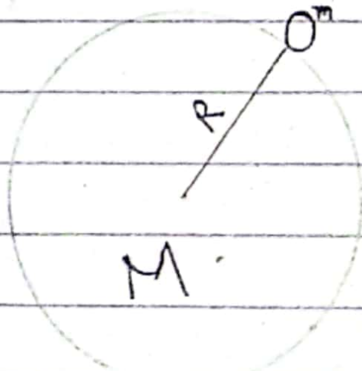
Combining eq^n(i) and eq^n(ii)

$$F \propto \frac{m_1 m_2}{d^2}$$

$$F = \frac{G m_1 m_2}{d^2} \text{ ---- (iii)}$$

Where G is proportionality Constant Called Universal Gravitational Constant and it's value is 6.67×10^{-11} $Nm^2 Kg^{-2}$.

Acceleration due to Gravity:-



Let us consider a body of mass 'm' lies on the surface of earth having mass 'M' and radius 'R'.

Then,

According to Newton's law of Gravitation, the force of attraction between earth and body is Given by;

$$F = \frac{GMm}{R^2} \quad \text{------(i)}$$

If g be the acceleration due to gravity then, according to Newton's second law of motion, The force acting on the body is given by;

$$F = mg \quad \text{----(ii)}$$

From eqn (i) and (ii), we get;

$$\cancel{m}g = \frac{GM\cancel{m}}{R^2}$$

$$\Rightarrow \boxed{g = \frac{GM}{R^2}} \quad \text{----(iii)}$$

This is required expression for acceleration due to gravity and relation shows that acceleration due to gravity is independent on the mass of body.

Variation of acceleration due to gravity:-

[1] Due to shape of earth:

↳ we know;

$$g = \frac{GM}{R^2}$$

$$\Rightarrow g \propto \frac{1}{R^2}$$

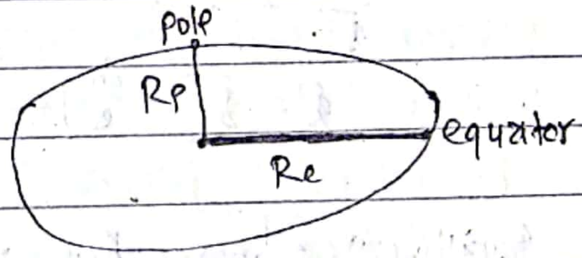


fig: Variation of g due to shape of earth

Since pole radius (Rp) is smaller than equator radius (Re). So, acceleration due to gravity at pole is greater than that a equator. i.e. $\boxed{g_p > g_e \text{ when } R_e > R_p}$.

[2] Due to height (Altitude):

↳ Let 'M' be the mass of earth and R be its radius. Also Let P be any point on the earth surface. So acceleration due to gravity at point P is given by:

$$g = \frac{GM}{R^2} \quad \text{---- (i)}$$

fig: variation of g due to height

Again, Let Q be any point at height 'h' from earth surface. So acceleration due to gravity at point Q is given by:

$$g' = \frac{GM}{(R+h)^2} \quad \text{---- (ii)}$$

Dividing eqn (ii) by eqn (i)

$$\frac{g'}{g} = \frac{R^2}{(R+h)^2}$$

$$\frac{g'}{g} = \frac{R^2}{\left[R\left(1+\frac{h}{R}\right)\right]^2}$$

$$\frac{g'}{g} = \frac{1}{\left(1+\frac{h}{R}\right)^2}$$

$$\Rightarrow \frac{g'}{g} = \left(1+\frac{h}{R}\right)^{-2}$$

Using binomial expansion and neglecting higher power term

$$\frac{g'}{g} = 1 - \frac{2h}{R}$$

$$g' = g\left[1 - \frac{2h}{R}\right] \quad \text{---- (iii)}$$

Equation (iii) shows that acceleration due to gravity goes on decreasing when height increase.

[3] Due to depth:

↳ Let 'M' be the mass of earth, 'R' be it's radius and 'ρ' be the density of earth. Also let 'P' be any point on the earth surface. So, acceleration due to gravity at point 'P' is given by:

$$g = \frac{GM}{R^2}$$

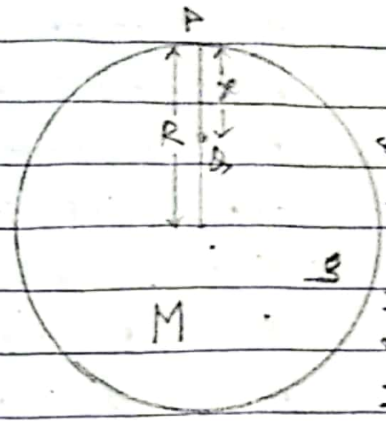


fig: Variation of g due to depth

or, $$g = \frac{G\frac{4}{3}\pi R^3 \rho}{R^2} \quad [\because \rho = \frac{m}{v}]$$

or, $$g = \frac{4}{3}\pi G R \rho \text{ ----- (i)}$$

Again, Let, B be any point at depth 'x' from earth surface then, acceleration due to gravity at point 'B' is given by

$$g' = \frac{4}{3}\pi G(R-x)\rho \text{ ----- (ii)}$$

Dividing eqⁿ (ii) by eqⁿ (i)

$$\frac{g'}{g} = \frac{R-x}{R} = \frac{R(1-\frac{x}{R})}{R}$$

$$\therefore g' = g\left[1-\frac{x}{R}\right] \text{ ----- (iii)}$$

eqⁿ (iii) shows that acceleration due to gravity decreases when we goes inner to the earth surface.

$\boxed{\text{At Centre, } x = R}$

$$\therefore g' = g\left[1-\frac{R}{R}\right] = g[1-1] = 0$$

Thus, In centre of earth acceleration due to gravity is zero.

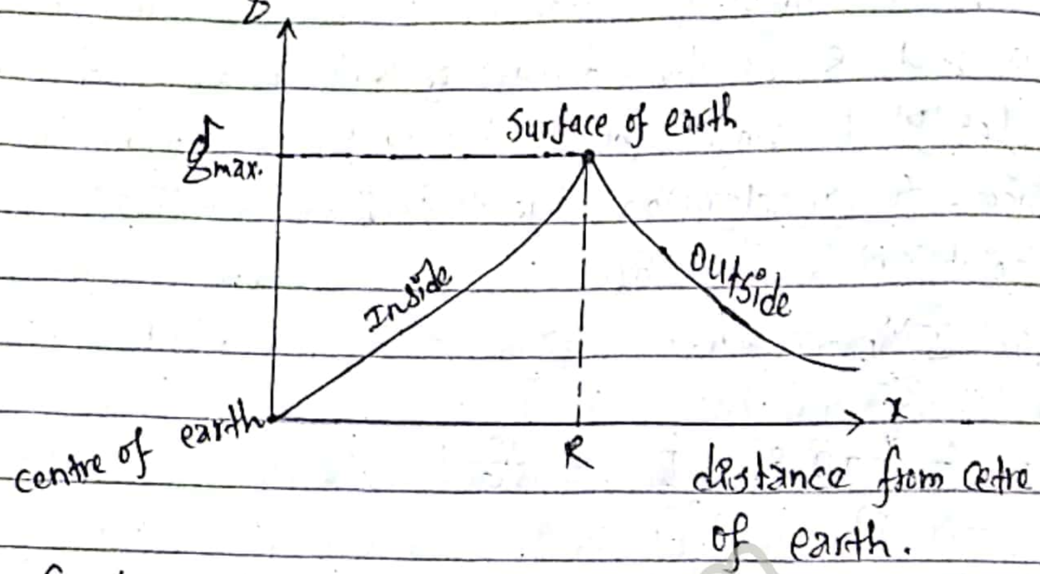


fig: Gaphical representation of variation of 'g' inside and outside the earth.

Gravitational Field :-

↳ The space area presence of gravitational force between particles is known as gravitational field.

Gravitational Field Intensity:-

↳ The force experienced by a unit test mass placed at that point is called gravitational field intensity.

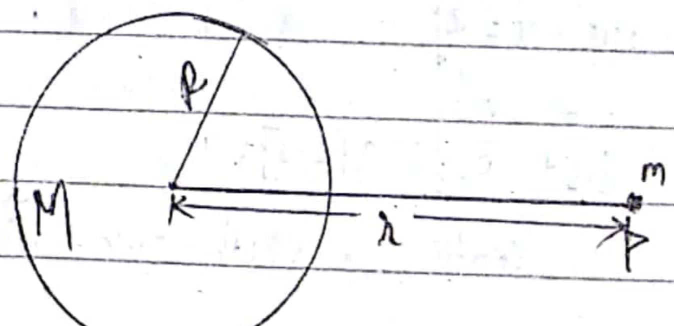


fig: gravitational field intensity

↳ Let us consider a body of mass 'm' is at distance 'r' from the centre of earth having mass 'M' and radius 'R'. So, the gravitational force between earth and body is given by;

$$F = \frac{GMm}{r^2}$$

Since, gravitational field intensity is given by;

$$E = \frac{F}{m}$$

$$\therefore E = \frac{GM}{r^2}$$

Hence, gravitational field intensity is force experienced by unit mass.

☆ On the surface of earth gravitational force intensity is given by:

$$E = \frac{GM}{R^2} = g$$

Hence, On the earth surface acceleration due to gravity is equal to gravitational force intensity.

Gravitational potential:-

↳ Gravitational potential at a point is defined as "amount of work done on bringing unit mass from infinity(∞) to that point."

↳ Let, 'p' be any point at distance 'r' from the centre of earth having mass 'M' and radius 'R', about which gravitational potential is to be determined.

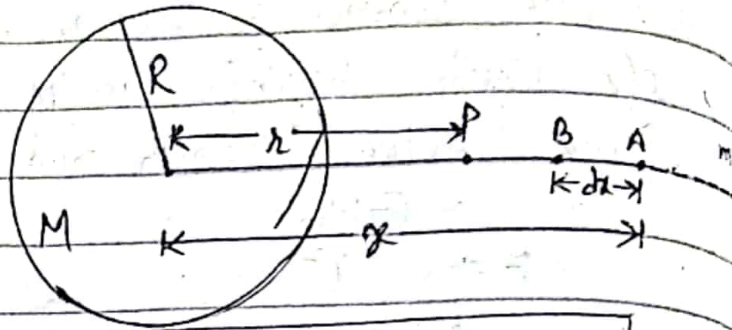


Fig: Gravitational potential

Suppose the unit mass is infinity(∞) distance from earth and at any instant it reach at point 'A' at distance 'x' from centre of earth. So, the gravitational force between earth and unit mass at point 'A' is given by;

$$F = \frac{GM.1}{x^2}$$

$$\Rightarrow F = \frac{GM}{x^2} \text{ ------ (i)}$$

Suppose the unit mass is further displace by small distance 'dx' towards the earth. So, small amout of workdone is given by; $dW = F.dx$

$$\Rightarrow dW = \frac{GM}{x^2}.dx \text{ ---- (ii)}$$

Then, total amount of workdone on bringing unit mass from '∞' to point 'p' is given by;

$$\Rightarrow W = \int_{\infty}^{r} \frac{GM}{x^2} dx = GM \int_{\infty}^{r} x^{-2} dx = GM\left[\frac{x^{-1}}{-1}\right]_{\infty}^{r} = -GM\left[\frac{1}{x}\right]_{\infty}^{r}$$

$$\text{or, } W = -GM\left[\frac{1}{r} - \frac{1}{\infty}\right] = -\frac{GM}{r}$$

So, gravitational potential, $\boxed{V = -\frac{GM}{r}}$ #

Gravitational potential energy;

↳ Gravitational potential energy at a point is defined as, "amount of work done on bringing mass 'm' of any body from infinity(∞) to that point".

↳ Let, 'p' be any point at distance 'r' from the earth having mass 'M' and radius 'R' about which gravitational potential energy is to be determined,

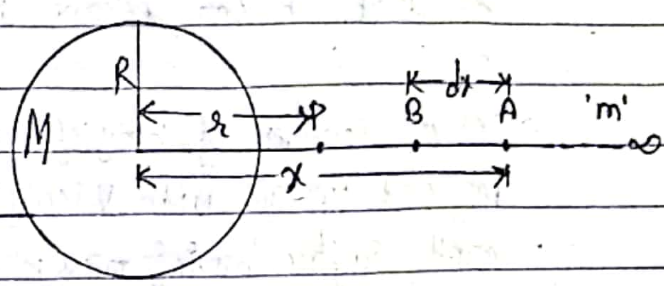


Fig: gravitational potential energy

Suppose the 'm' mass is '∞' distance from earth and ~~at~~ any instant it reach at point 'A' at distance 'x' from centre of earth. So, gravitational force between earth and 'm' mass at point 'A' is given by;

$$F = \frac{GMm}{x^2} \quad \text{----- (i)}$$

Suppose the mass 'm' is further displace by small distance 'dx' towards the earth. So, small amout of workdone is given by;

$$dW = F \cdot dx$$

$$\Rightarrow dW = \frac{GMm}{x^2} \cdot dx \quad \text{----- (ii)}$$

Then, total amount of work done on bringing 'm' mass from '∞' to 'p' is given by;

$$\Rightarrow W = \int_{\infty}^{r} \frac{GMm}{x^2} dx = GMm \int_{\infty}^{r} x^{-2} dx = GMm \left[\frac{x^{-2+1}}{-2+1}\right]_{\infty}^{r} = -GMm\left[\frac{1}{x}\right]_{\infty}^{r}$$

$$\text{or, } W = -GMm\left[\frac{1}{r} - \frac{1}{\infty}\right] = \frac{-GMm}{r}$$

So, gravitational potential energy, $\boxed{U = -\frac{GMm}{r}}$

V.V.Imp

Escape Velocity

↳ The minimum velocity with which a body must be projected upward from the Surface of earth to overcome it's gravitational pull so that it can escape into space is called escape velocity.

↳ Let us consider of body of mass'm' is projected upward with velocity 'v' from the earth surface having mass'M' and radius'R'.
Then, kinetic energy of the body is given by;

$K.E. = \frac{1}{2}mv^2$.

Also suppose at any instant the body reach at point 'A' at distance 'x' from the centre of earth..

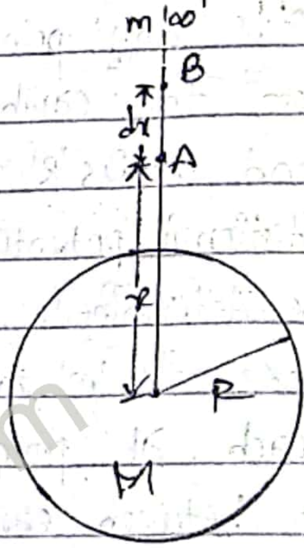


fig: escape velocity

then, gravitational force between earth and body is given by; (At point A)

$$F = \frac{GMm}{x^2}$$

Also, the body is further displaced upward by small distance 'dx'. The small amount of workdone is given by;

$$dW = F.dx$$

Then, total amount of workdone is given by;

$$W = \int_R^\infty dW = \int_R^\infty \frac{GMm}{x^2}dx = GMm\int_R^\infty x^{-2}dx = GMm\left[\frac{x^{-2+1}}{-2+1}\right]_R^\infty$$

$$\text{or, } W = -GMm\left[\frac{1}{x}\right]_R^\infty = -GMm\left[\frac{1}{\infty} - \frac{1}{R}\right] = \frac{GMm}{R}$$

From work energy theorem, work done = change in K.E.

$$\frac{GM\cancel{m}}{R} = \frac{1}{2}\cancel{m}v^2$$

$$v = \sqrt{\frac{2GM}{R^2}.R}$$

$$\therefore \boxed{v = \sqrt{2gR}} \quad \left[\because g = \frac{GM}{R^2}\right]$$

This is required expression for escape velocity.

For earth; $g = 9.8\ ms^{-2}$, $R = 6400\ km = 6.4 \times 10^6 m$

Then, escape velocity $(V_e) = \sqrt{2 \times 9.8 \times 6.4 \times 10^6} = \sqrt{125.44 \times 10^6} = 11.2 \times 10^3 m/s = 11.2\ km/s$

Satellite:-

↳ The body which revolve around the planet is called satellite. The path of the satellite around the planet is called orbit of satellite. The velocity of satellite on it's orbit is called orbital velocity.

Expression for orbital Velocity:-

↳ Let us consider a satellite of mass 'm' is revolving around the earth of mass 'M' & radius 'R' on it's orbit of radius 'r'. The gravitational force between earth & satellite is given by;

$$F = \frac{GMm}{r^2} \quad \text{---} (i)$$

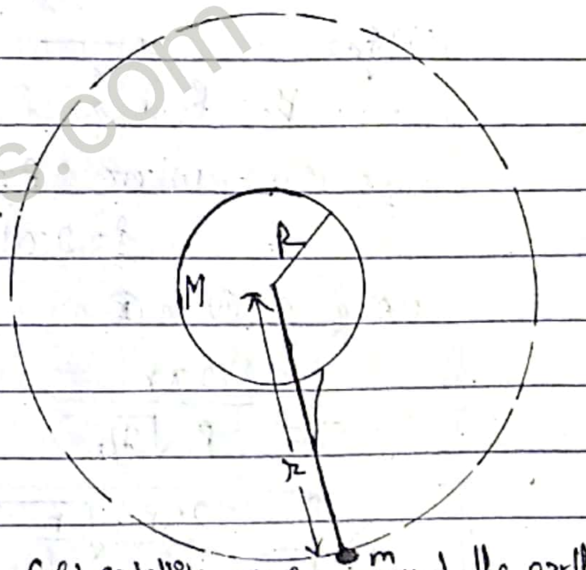


fig: satellite moving around the earth.

Also Let 'V_o' be the orbital velocity of satellite then centripetal force experienced by satellite is given by;

$$F_c = \frac{mV_o^2}{r} \quad \text{---} (ii)$$

so, gravitational force provides necessary centripetal force,

i.e., $F = F_c$

or, $\frac{GM\cancel{m}}{r^{\cancel{2}}} = \frac{\cancel{m}V_o^2}{\cancel{r}}$

or, $V_o = \sqrt{\frac{GM}{r}} \quad \text{---} (iii)$

or, $V_o = \sqrt{\frac{GM}{R^2} \times \frac{R^2}{r}} = \sqrt{g\frac{R^2}{r}}$

$$\therefore \boxed{V_o = R\sqrt{\frac{g}{r}}} \quad \text{---} (iv)$$

Thus, eqn (iii) and (iv) are required expression for orbital velocity.

Time period of satellite :-

↳ The time taken by satellite to complete one revolution around the earth is called time period.

If 'T' be the time period of satellite,

Then, $$T = \frac{2\pi r}{v_0} \quad \text{---} (I)$$

where, r = radius of orbit
v_0 = orbital velocity

Again,
$$v_0 = R\sqrt{\frac{g}{r}} \quad \text{---} (II)$$

where, R = radius of earth
g = acceleration due to gravity

Using eqn (I) in (II)

$$T = \frac{2\pi r}{R\sqrt{g/r}}$$

$$\therefore \boxed{T = \frac{2\pi r}{R}\sqrt{\frac{r}{g}}}$$ This is required expression for time period of satellite.

Height of satellite :-

↳ The distance of satellite from earth surface is called height of satellite.

Let, T = time period of satellite
r = radius of orbit
R = radius of earth
g = acceleration due to gravity

height of satellite $(h) = (r - R)$

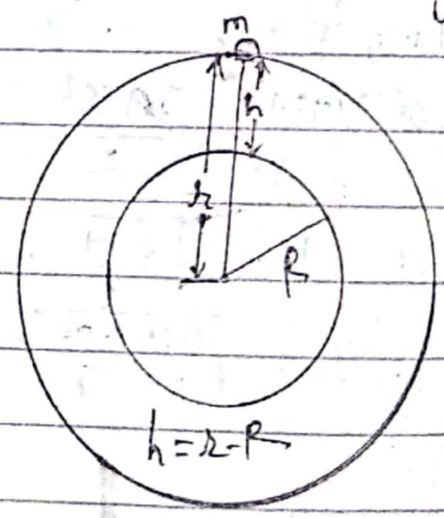


fig: Height of satellite

We have,

$$T = \frac{2\pi r}{R}\sqrt{\frac{r}{g}}$$

Since, $r = R+h$

$$\therefore T = \frac{2\pi(R+h)}{R}\sqrt{\frac{R+h}{g}}$$

$$\text{or, } T^2 = \frac{4\pi^2(R+h)^3}{R^2 g}$$

$$\text{or, } (R+h)^3 = \frac{T^2R^2g}{4\pi^2}$$

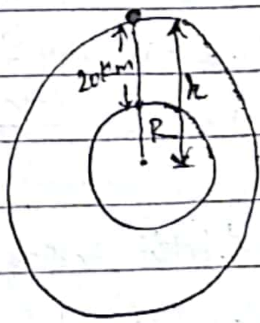
$$(R+h) = \left(\frac{T^2R^2g}{4\pi^2}\right)^{1/3}$$

$$\Rightarrow h = \left(\frac{T^2R^2g}{4\pi^2}\right)^{1/3} - R$$

Which is required expression for height of satellite.

L.Q. 1 [A] Calculate the period of revolution of a satellite revolving at a distance of 20 km above the surface of the earth (Radius of the earth = 6400 km, g = 10 m/s)

☆ Solution:-



height of Satellite $(h) = 20$ km

Radius of earth $(R) = 6400 \text{ km} = 6.4 \times 10^6$ m.

radius of orbit $(r) = R+h = 6420$ km

$= 6.42 \times 10^6$ m.

Time period of Satellite $(T) = ?$

Now, we have,

$$T = \frac{2\pi r}{v_0} = \frac{2\pi r}{R\sqrt{g/r}} = \frac{2(3.14).6.42\times10^6}{6.4\times10^6\sqrt{\frac{10}{6.42\times10^6}}} \qquad \left[\because v_0 = R\sqrt{\frac{g}{r}}\right]$$

$$\text{or, } T = \frac{40.3176}{6.4\times 1.248\times10^{-3}} = 5048 \text{ sec.}$$

Hence, time period of satellite is 5048 sec #

Energy of Satellite:-

↳ Let us consider a satellite of mass 'm' is revolving around the earth of mass 'M' & radius 'R' on it's orbit of radius 'r'.

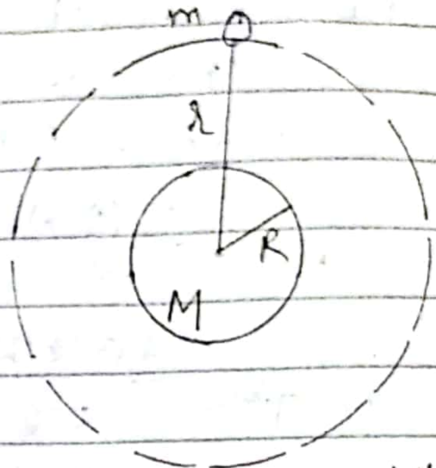


fig: satellite revolve around the earth

Now,
The gravitational force between earth & Satellite is given by;

$$F = \frac{GMm}{r^2} \quad \text{---} (i)$$

The gravitational force provides the necessary centripetal force.

i.e., $F = F_c$

or, $\frac{GMm}{r^2} = \frac{mv^2}{r}$

or, $mv^2 = \frac{GMm}{r}$

or, $\frac{1}{2}mv^2 = \frac{GMm}{2r}$

$\therefore K.E. = \frac{GMm}{2r}$ —— (ii)

→ Again, potential energy at a point on it's orbit is given by;

$$P.E. = \frac{-GMm}{r} \quad \text{---} (iii)$$

Then, total Energy of the satellite is given by;

$$E = K.E. + P.E. = \frac{GMm}{2r} - \frac{GMm}{r} = \frac{GMm - 2GMm}{2r}$$

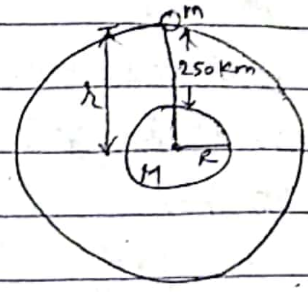
$\therefore \boxed{E = \frac{-GMm}{2r}}$ this is required expression and negative sign shows that satellite is attracted towards earth.

Geostationary Satellite:-

↳ The satellite which always appears at same point from any point on the earth and whose time period equals time period of earth is called geostationary satellite. The orbit of the geostationary satellite is called parking orbit.

[LQ.1(B)] A remote sensing satellite of the earth revolves in a circular orbit at a height of 250 km above the earth's surface. What is the orbital speed and period of revolution on satellite.

A Soln:-



We have, Radius of earth $(R) = 6400km = 6.4 \times 10^6 m$.

height of satellite $(h) = 250 km$

radius of orbit $(r) = R+h = (6400+250) Km = 6.65 \times 10^6 m$.

Now,

orbital velocity $(V_0) = R\sqrt{g/r}$

$= 6.4 \times 10^6 \times \sqrt{10/6.65 \times 10^6}$

$= 6.4 \times 10^6 \times 1.22 \times 10^{-3}$

$\therefore V_0 = 7848.18 \ m/sec$

Again, Time period $(T) = \frac{2\pi r}{V_0} = \frac{2 \times (3.14) . 6.65 \times 10^6}{7848.18} = 5321 \ sec.$

Thus, orbital speed is 7848.18 m/s and time period is 5321 seconds. #

[LQ.1(C)] An artificial satellite revolves round the earth in 2.5 hours in a circular orbit. Find the height of the satellite above the earth assuming earth as a sphare of radius 6370 Km.

A Soln:-

$T = 2.5 \ hrs = 2.5 \times 60 \times 60 = 9000 \ sec$

$h = ?, R = 6370 \ km = 6.3 \times 10^6 m.$

$r = R+h \ / \ h = r - R$

We have, $T = 9000$ sec.

or, $\frac{2\pi r}{R\sqrt{g/r}} = 9000$

or, $\frac{3.14(R+h)}{R\sqrt{10/(R+h)}} = 4500$

or, $R+h = 1433.12R\sqrt{\frac{10}{R+h}}$

or, $(R+h)^2 = 2053835.855 R^2 \times \frac{10}{R+h}$

or, $(R+h)^3 = 20538358.55 R^2$

or, $(6.37 \times 10^6 + h)^3 = 20538358.55 (6.37 \times 10^6)^2$

or, $(6.37 \times 10^6 + h)^3 = 8.3 \times 10^{20}$

$\Rightarrow 6.37 \times 10^6 + h = 9410546.941$

$\therefore \boxed{h = 3040546.941 \ m. = 3040 \ km}$

Hence, the period of revolution on satellite is 2.5 hrs where, height of satellite will be 3040 km. #

LQ.1[D] Obtain the value of g from the motion of moon assuming that its period of rotation round the earth is 27 days 8 hours & the radius of its orbit is 60.1 times the radius of the earth.

A Soln:- given,

$T = 27$ days 8 hours $= 27$ days $+ 8$ hours $= (27\times24\times60\times60) + (8\times60\times60) = 2361600$ sec.

$r = 60.1R$, $g = ?$

We have,

$$T = \frac{2\pi r}{v_0}$$

or, $2361600 = \dfrac{2(3.14)(60.1R)}{R\sqrt{g/r}}$

or, $376050.9554 = \dfrac{60.1R}{R\sqrt{g/r}}$

or, $6257.087444 = \sqrt{r/g}$

or, $39151143.29 = r/g = \dfrac{60.1R}{g}$

or, $651433.3326 = \dfrac{6400000}{g}$

$\Rightarrow \boxed{g = 9.82\ m/s^2}$

Thus, the value of g is $9.82\ m/s^2$. #

LQ.1[E] The period of moon revolving under the gravitational force of the earth is 27.3 days. Find the distance of the moon from the centre of the earth if the mass of earth is 5.97×10^{24} kg.

A Soln

Let, period of moon → T, Mass of earth → M, Radius of earth → R, radius of orbit → r, then,

$T = 27.3$ days $= (27.3\times24\times60\times60)$ sec $= 2358720$ sec, $r = ?$, $M = 5.97\times10^{24}$ kg, $R = 6400$ km $= 6.4\times10^6$ m.

We have,

$$T = \frac{2\pi r}{R\sqrt{g/r}}$$

$$\boxed{g = \frac{GM}{R^2} = \frac{6.67\times10^{-11}\times5.97\times10^{24}}{(6.4\times10^6)^2} = 9.72\ m/s^2}$$

or, $2358720 = \dfrac{2(3.14)r}{6400000\sqrt{9.72/r}}$

or, $2.4\times10^{12} = r\sqrt{\dfrac{r}{9.72}}$

or, $5.77821\times10^{24} = \dfrac{r^3}{9.72}$

or, $r^3 = 5.77821\times10^{25}$

$\Rightarrow \boxed{r = 3.83\times10^8\ m}$

Hence, height of moon is 3.83×10^8 m. #

L.Q. 1 [F] An earth satellite moves in a circular orbit with a speed of 6.2 kms⁻¹. Find the time of one revolution and it's centripetal acceleration.

Sol^n:- $v_0 = 6.2$ Km/s $= 6200$ m/s, $T = ?$, $\alpha = ?$, $R = 6400$ km $= 6400000$ m.

Now,

$v_0 = R\sqrt{g/R+h} = 6200$

or, $6400000 \times \sqrt{\frac{10}{6400000+h}} = 6200$

or, $9.69 \times 10^{-4} = \sqrt{\frac{10}{6400000+h}}$

or, $6400000 + h = \frac{10}{9.5385 \times 10^{-7}}$

$\Rightarrow \boxed{h = 4255301 \text{ m}}$

Again,

$T = \frac{2\pi r}{v_0}$

$= \frac{2(3.14).(6.4\times10^6 + 4255301)}{6200}$

$= 10802.60931$ sec

$\therefore \boxed{T = 3 \text{ hrs}}$

Again

$\alpha = \omega^2 r$

$= \left(\frac{v_0}{r}\right)^2 \times r$

$= \frac{v_0^2}{r}$

$= \frac{(6200)^2}{4255301 + 6400000}$

$\therefore \boxed{\alpha = 3.6 \text{ m/s}^2}$

Hence, time of 1 revolution is 3 hrs & centripetal acceleration is 3.6 m/s². #

L.Q. 1 [G] What is is the period of revolution of a satellite of mass 'm' that orbits the earth in a circular path of radius 7880 km about 1500 km above the surface of the earth.

Sol^n:-

$T = ?$, $R = 7880$ km $= 7.88 \times 10^6$ m, $h = 1500$ km, $r = R + h = 9.38 \times 10^6$ m.

Now,

orbital velocity,

$v_0 = R\sqrt{g/r}$

$= 7.88 \times 10^6 \sqrt{\frac{10}{9.38 \times 10^6}}$

$= 7.88 \times 10^{6-3} \times \sqrt{\frac{10}{9.38}}$

$\Rightarrow$

$\boxed{v_0 = 8136.259625 \text{ m/s}}$

Again,

Time period, $(T) = \frac{2\pi r}{v_0}$

$= \frac{2(3.14).(9.38\times10^6)}{8136.259625}$

$= 7239.98529$ sec

$\Rightarrow \boxed{T = 2 \text{ hours}}$

Hence, period of revolution is 2 hrs. #

$$\boxed{\rho = \frac{m}{V}}$$

may be
[Imp]

LQ. [2(A)] A man can jump 1.5m on earth. Calculate the approximate height he might be able to jump

LQ. 2[i] Calculate the points along a line joining the centres of earth and moon where there is no gravitational force. [$M_e = 6\times10^{24}$ kg, $M_m = 7.4\times10^{22}$ kg, $d = 3.8\times10^{8}$ m].

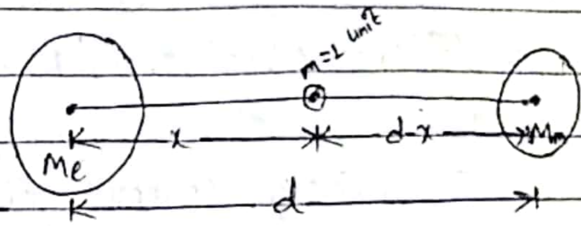
Ans:- Soln:-

Let, point mass $= m = 1$ unit.

x is distance from earth to point &

$(d-x)$ is distance from moon to point mass.

where, d is distance between centres of moon & earth.



Now,

Gravitational force between earth & unit mass;

$$F_1 = \frac{GM_e.1}{x^2} = \frac{GM_e}{x^2}$$

Gravitational force between moon & unit mass;

$$F_2 = \frac{GM_m.1}{(d-x)^2} = \frac{GM_m}{(d-x)^2}$$

There is no gravitational force;

i.e, $F_1 = F_2$

or, $\dfrac{GM_e}{x^2} = \dfrac{GM_m}{(d-x)^2}$

or, $\dfrac{6\times10^{24}}{x^2} = \dfrac{7.2\times10^{22}}{(3.8\times10^{8}-x)^2}$

or, $\left(\dfrac{81.08}{x}\right)^2 = \left(\dfrac{1}{3.8\times10^{8}-x}\right)^2$

$\Rightarrow \dfrac{81.08}{x} = \dfrac{1}{3.8\times10^{8}-x}$

or, $x = 3.08\times10^{10} - 81.08x$

or, $(1+81.08)x = 3.08\times10^{10}$

$\Rightarrow \boxed{x = 3.75\times10^{8}\text{ m.}}$

Hence, the point is 3.75×10^{8} m away from centre of earth. #

LQ. 3[A] Taking the earth to be uniform sphare of radius 6400 km, Calculate the total energy needed to raise a satellite of mass 1000 kg to a height of 600 km above the ground & to set it into Circular orbit at that altitude.

A) Soln:- $R = 6400\ km = 6.4\times10^{6}m.$, $m = 1000\ kg$, $h = 600\ km$, $r = R+h = (6400+600)km = 7\times10^{6}m.$

Now,

Total energy = Increase in P.E. + K.E.

$$= \frac{-GMm}{r} - \left(\frac{-GMm}{R}\right) + \frac{1}{2}mv_0^2$$

or, $GMm\left[\frac{1}{R}-\frac{1}{r}\right] + \frac{m}{2}\left[R^2\frac{g}{r}\right] = 6.67\times10^{-11}\times6\times10^{24}\times1000\left[\frac{1}{6.4\times10^{6}}-\frac{1}{7\times10^{6}}\right] + \frac{1000}{2}\left[\frac{(6.4\times10^{6})^2\times10}{7\times10^{6}}\right]$

or, $(4.002\times10^{17}\times1.339\times10^{-6}) + 2.92\times10^{10} = 5.3598\times10^{2} + 2.92\times10^{10}$

$\Rightarrow \boxed{E = 3.46\times10^{10}J}$

thus, total needed energy is $3.46\times10^{10}J$. #

LQ. 3[B] Taking the earth be uniform sphare of radius 6400 km & the value of g at the surface to be 10 m/s². Calculate the total energy needed to raise a satellite of mass 2000 kg to a height of 800 km above the ground and to set it into circular orbit at that altitude.

A) Soln:- $R = 6400\ km = 6.4\times10^{6}m.$, $g = 10m/s^2$, $m = 2000\ kg$, $h = 800\ km$, $r = R+h = 7.2\times10^{6}m.$

Now,

Total Energy(E) = Increase in P.E. + K.E. $= \frac{-GMm}{r} - \left[\frac{-GMm}{R}\right] + \frac{1}{2}mv_0^2$

or, $\frac{1}{2}\cdot2000\cdot R^2\times\frac{g}{r} + GMm\left[\frac{1}{R}-\frac{1}{r}\right] = \frac{1000[6.4\times10^{6}]^2\times10}{7.2\times10^{6}} + \frac{6.67\times10^{-11}\times6\times10^{24}\times2000}{1}\left[\frac{1}{R}-\frac{1}{r}\right]$

or, $5.689\times10^{10} + [8.004\times10^{17}(1.736\times10^{-6})] = (5.689\times10^{10} + 1.38958\times10^{10})J$

$\Rightarrow \boxed{E = 7.07\times10^{10}J}$

Hence, total needed energy is $7.07\times10^{10}J$ #

LQ. 3[c] A 200 kg. satellite is lifted to an orbit of 2.2×10^4 km radius. If the radius and mass of the earth are 6.37×10^6 m and 5.98×10^{24} kg respectively, how much additional potential energy is required to lift the satellite?

Soln:- $m = 200$ kg, $r = 2.2\times10^4$ km $= 2.2\times10^7$ m, $R = 6.37\times10^6$ m, $M = 5.98\times10^{24}$ kg

Now,

additional P.E. = P.E. on orbit − P.E. on earth

$$= -\frac{GMm}{r} - \left(-\frac{GMm}{R}\right)$$

or, $$GMm\left[\frac{1}{R} - \frac{1}{r}\right] = 6.67\times10^{-11}\times5.98\times10^{24}\times200\left[\frac{1}{6.37\times10^6} - \frac{1}{2.2\times10^7}\right]$$

$$= 79.7732\times10^{15}\left[1.11\times10^{-7}\right]$$

$\Rightarrow$ $\boxed{\text{Additional P.E.} = 8.89\times10^9 \text{ J}}$ #