CHAPTER (08)

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ELASTICITY:-

Elasticity and plasticity

↳ The property of the body which regains its original Confugiration after removing deforming force is called elasticity.
For example :- The property of the body like quartz, phosphor, bronze etc are perfectly elastic in nature.

↳ the property of the body which doesn't regains its orginal Confugiration after removing deforming force is called plasticity.
For example :- The property of body like paraffin wax, wet clay etc are perfectly plasticity in nature.

Deforming Force :-

↳ The force which is responsible for change in Confugiration is called deforming force.

Restoring Force :-

↳ The force which is responsible for regain its original position is called Restoring force.

Stress

↳ The deforming force or restoring force per unit area is called stress.

i.e., $\text{Stress} = \dfrac{\text{Deforming force or Restoring force}}{\text{Area}}$

$\Rightarrow \boxed{\text{stress} = \dfrac{F}{A}}$

It is a scalar quantity and it's SI unit is Nm^{-2}. It's dimensional formula is $[ML^{-1}T^{-2}]$

Types of Stress:-

[1] Normal Stress:-

↳ The deforming force per unit area acting perpendicular to it is called normal stress.

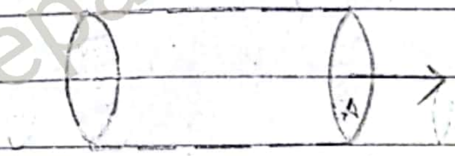

fig: Normal Stress

[2] Bulk stress/volume stress:-

↳ When a body is immersed in liquid, the liquid molecules strikes to the surface from all direction and such force applied by liquid molecules per unit area is called bulk ~~molecul~~ stress.

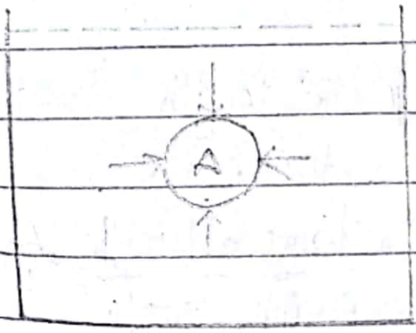

fig: Bulk stress

[3] Shear stress/Tangential stress:-

↳ The stress in which force is applied tangentially in two opposite faces and which change only shape but not volume is called shear stress.

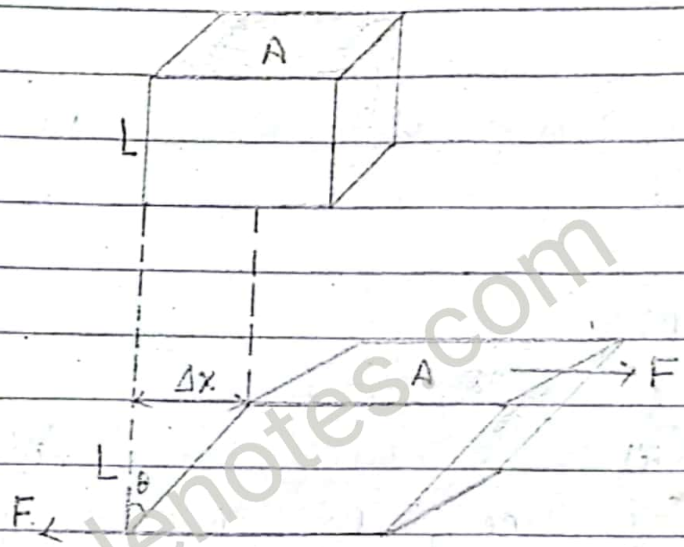

fig: Shear/Tangential stress

Strain:-

↳ The ratio of change in configuration to the original configuration is called strain.

Types of Strain:-

[1] Longitudinal Strain:-

↳ The ratio of change in length to the original length is called longitudinal strain.

i.e., $\text{Longitudinal strain} = \dfrac{\text{Change in length}}{\text{Original length}} = \dfrac{\Delta L}{L} = \dfrac{l}{L}$

fig: Longitudinal strain.

[2] Volumetric strain:

↳ The ratio of change in Volume to the original Volume is called Volumetric Strain.

i.e., Volumetric Strain $= \dfrac{\text{Change in Volume}}{\text{Original volume}} = \dfrac{\Delta V}{V}$

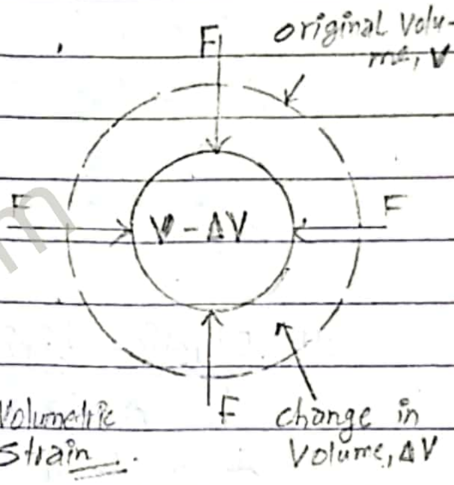

fig: Volumetric Strain

[3] Lateral strain :-

↳ The ratio of change in diameter to the original diameter is called Lateral Strain.

i.e., Lateral Strain $= \dfrac{\text{Change in diameter}}{\text{original diameter}} = \dfrac{\Delta d}{d}$

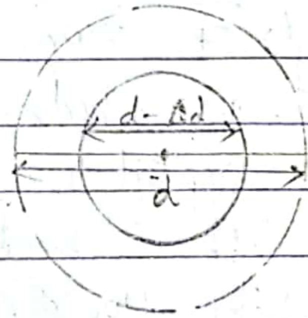

fig: Lateral Strain

[4] Shear Strain:

↳ When a body is subjected to shear Stress then angle between vertical faces at displaced position and original position is called Shear stress.

i.e, shear strain = angle of deviation from the original position = θ

From figure,

$\tan\theta = \dfrac{\Delta x}{x}$

If $\theta << 1$,

then, $\tan\theta \simeq \theta$

$$\therefore \boxed{\theta = \dfrac{\Delta x}{x}}$$

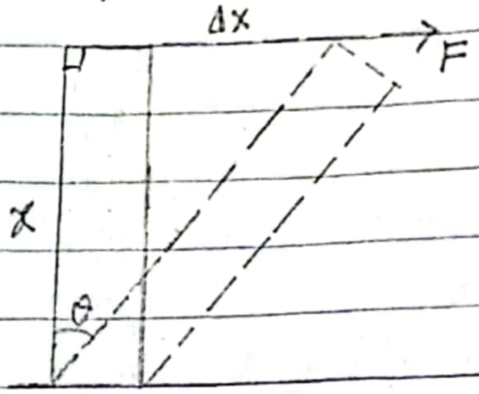

fig: Shear strain.

Elastic Limit:-

↳ The maximum Value of deforming force which can be applied to a body up to which it remains plastic is called elastic limit.

Hooke's law:-

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↳ It state that "within elastic limit the elongation produce in a body is directly proportional to the deforming force applied to it.

If 'F' is the deforming force applied for elongation 'e'. then by Hooke's law;

$$e \propto F$$

or, $\dfrac{e}{A} \propto \dfrac{F}{A}$

or, $\dfrac{F}{A} \propto \dfrac{e}{L} \cdot \dfrac{L}{A}$

Since, $\dfrac{F}{A}$ = Stress, $\dfrac{e}{L}$ = Strain & $\dfrac{L}{A}$ = Constant for given w

Thus, Hooke's law can be restate as "within elastic limit stress is directly proportional to strain."

Experimental verification of Hooke's law :-

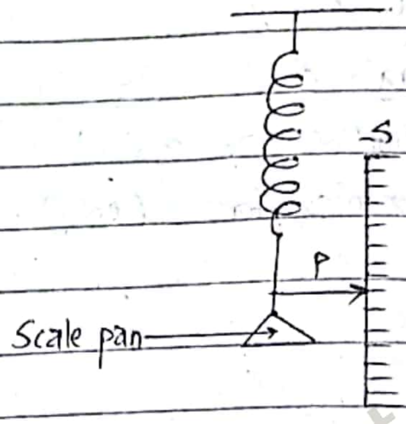

fig: Experimental arrangement of verification of Hooke's law.

In this experiment, spring is suspended vertically as shown in figure above. Scale pan and pointer are attatched to the lower end of spring. Scale pan is attatched in order to keep load and pointer is attatched in order to check scale reading.

When the scale pan is empty, note the reading of pointer on the scale and at some weight on the scale pan. then, again note the reading of pointer on the scale.

The difference in two reading will give the elongation in spring for the weight on the scale pan. Go on adding the weight on the scale pan in the step by step and note the corresponding elongation in the spring. Finally if we plot the graph between weight on scale pan and elongation in the spring then we obtained a straight line passing through origin as shown in figure below:-

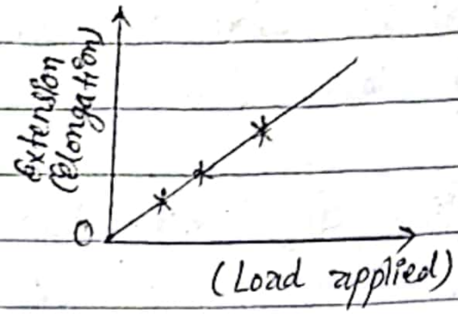

fig: stress-strain graph for Hooke's law

From above graph, elongation $\propto$ load applied

i.e., elongation $\propto$ deforming force $(e \propto F)$

Which is Hooke's law.

Types of modulus of elasticity:-

[1] Young's modulus of elasticity (Y):-

↳ The ratio of normal stress to the Longitudinal strain is called Young's modulus. It's SI unit is Nm^{-2} or pa. #

i.e., $Y = \dfrac{\text{normal stress}}{\text{Longitudinal strain}}$

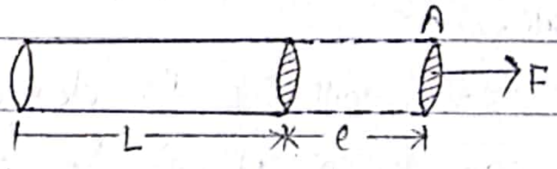

fig: Young's modulus

Let, L = original length of wire, e = elongation, A = cross sectional area and F = Applied deforming force.

Then, $Y = \dfrac{F/A}{e/L} = \dfrac{F}{A} \times \dfrac{L}{e} = \dfrac{FL}{eA}$.

[2] Bulk modulus of elasticity [B] or [K]

↳ The ratio of bulk stress to the volumetric strain is called Bulk modulus.

i.e., $B[K] = \dfrac{\text{Bulk stress}}{\text{Volumetric Strain}}$

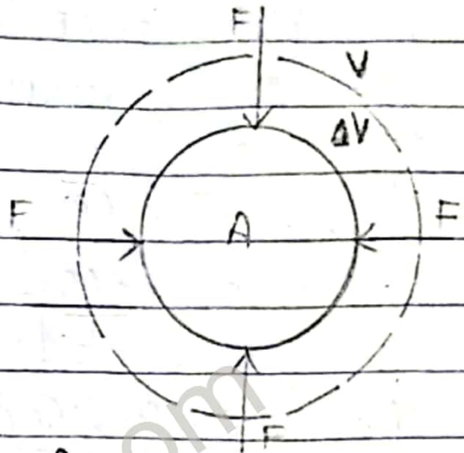

fig: Bulk modulus.

Let,
F = Applied deforming force
A = Cross sectional area, V = original Volume
ΔV = Change in Volume.

then,

$$K = \frac{F/A}{-\Delta V/V} = -\frac{F}{A} \times \frac{V}{\Delta V} = -\frac{FV}{A\Delta V}$$

[3] Shear modulus of elasticity [η]:-

↳ The ratio of shear stress to the shear strain is called Shear modulus.

i.e.,

$$\text{Shear modulus}(\eta) = \frac{F/A}{\theta} = \frac{F}{A} \times \frac{1}{\theta} = \frac{F}{A\theta}$$

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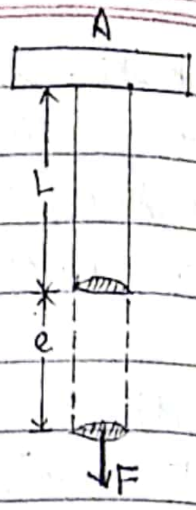
Energy stored in a stretched wire:-



↳ Let us consider a uniform metalic wire having initial length 'L' and cross sectional area 'A'. By applying force 'F', it is elongated by 'e'. then, Young's modulus of material is given by;

$$Y = \frac{F/A}{e/L} = \frac{FL}{eA}$$

$$\Rightarrow \boxed{F = \frac{YAe}{L}} \quad ----(i)$$

If the wire is stretched through a small length 'de' then small amount of workdone is given by;

$$dW = F.\,de$$
$$= \frac{YAe}{L}.\,de \quad —(ii)$$

Now, total amount of workdone on the wire when stretched from '0' to 'e' is given by;

$$W = \int_0^e dW = \int_0^e \frac{YAe}{L}\,de = \frac{YA}{L}\int_0^e e\,de = \frac{YA}{L}\left[\frac{e^2}{2}\right]_0^e = \frac{YA}{2L}\left[e^2 - 0\right]$$

$$\Rightarrow \boxed{W = \frac{1}{2}.\frac{YAe}{L}.\,e} \quad —(iii)$$

using eq^n (i) in eq^n (iii)

$$\Rightarrow \boxed{W = \frac{1}{2}F.e}$$

This workdone is store in the form of P.E.

$$\therefore \boxed{E = \frac{1}{2}F.e}$$

Again,

P.E. per unit volume (or Energy density) $= \frac{YAe^2}{2L} \times \frac{1}{\text{Volume}}$

$$= \frac{YAe^2}{2L} \times \frac{1}{A \times L}$$
$$= \frac{1}{2}.\,Y.\,\frac{e^2}{L^2}$$
$$= \frac{1}{2} \times \frac{\text{stress}}{\text{strain}} \times (\text{strain})^2$$

$$\therefore \boxed{\text{P.E. per unit volume} = \frac{1}{2}\,\text{Stress} \times \text{Strain}}$$

#

Numericals

Q.1[A] A Copper wire and steel wire of the same cross sectional area and of length 1m and 2m. respectively are connected end to end. A force is applied, which stretched their combined length by 1cm. Find how much each wire is elongated.

Soln:- Given;

length of Copper $(L_c) = 1m$, $Y_c = 1.2\times10^{11}$ N/m²

length of Steel $(L_s) = 2m$, $Y_s = 2\times10^{11}$ N/m²

$e_s + e_c = 0.01$ --- (i)

Now, $\frac{Y_s}{Y_c} = \frac{FL_s}{e_sA} / \frac{FL_c}{e_cA} = \frac{\cancel{F}L_s}{e_s\cancel{A}} \times \frac{e_c\cancel{A}}{\cancel{F}L_c}$

or, $\frac{\cancel{2}\times10^{11}}{1.2\times10^{11}} = \frac{\cancel{2}\times e_c}{e_s\times1}$

$\Rightarrow \boxed{e_s = 1.2e_c}$ — (ii)

Again, from (i)

or, $e_s + e_c = 0.01$

or, $1.2e_c + e_c = 0.01$ [$\because$ using eqn (ii)]

$\Rightarrow \boxed{e_c = 4.5\times10^{-3}\ m}$

putting the value of e_c in eqn (i)

or, $e_s + 4.5\times10^{-3} = 0.01$

$\Rightarrow \boxed{e_s = 5.5\times10^{-3}\ m}$

Q.2[A] The rubber cord of a Catapult has a cross-sectional area 1 mm² and total unstretched length 10 cm. It is stretched to 12 cm and then released to project a missile of mass 5g. Calculate the velocity of projection.

Soln:-

$A = 1\ mm^2 = 10^{-2}\ cm^2 = 10^{-6}\ m^2$, $L = 10cm = 0.1\ m$, $e = 12cm - 10cm = 2cm = 0.02\ m$.

$v = ?$, $m = 5g = 5\times10^{-3}\ kg$, $Y = 5\times10^{8}$ N/m²

Now, $Y = \frac{F/A}{e/L} = \frac{F}{A}\times\frac{L}{e}$

or, $5\times10^8 = \frac{F\times0.1}{10^{-6}\times0.02}$

$\Rightarrow \boxed{F = 10^2\ N}$

Again, $\frac{1}{\cancel{2}}mv^2 = \frac{Fe}{\cancel{2}}$

or, $5\times10^{-3}\cdot v^2 = \frac{10^2\times0.02}{1}$

$\Rightarrow \boxed{v = 20\ m/s}$

Thus, velocity of projection is 20 m/s.

Q.3[A] A wire of length 2.5 m and area of cross-setion $1\times10^{-6}\,m^2$ has a mass of 15 Kg hanging on it. What is the extension produced? How much is the energy stored in the extended when if Young's modulus of wire is $2\times10^{11}\,N/m^2$.

☆ Soln:-

$L = 2.5m$, $A = 1\times10^{-6}\,m^2$, $Y = 2\times10^{11}\,N/m^2$, $e = ?$, $E = ?$, $m = 15\,Kg$

Now;

$Y = \frac{F}{A}\times\frac{L}{e}$

or, $2\times10^{11} = \frac{mg\times2.5}{1\times10^{-6}\times e}$

or, $2e\times10^{5} = 150\times2.5$

$\Rightarrow \boxed{e = 18.75\times10^{-5}\,m}$

Again, Energy stored $(E) = \frac{Fe}{2} = \frac{150\times18.75\times10^{-5}}{2} = 0.14\,J$ #

Q. 3[B] A uniform steel wire of density $8000\,Kg/m^3$ weight 20g and is 2.5 m long. It lengthens by 1 mm when stretched by a force of 80N. Calculate the value of the Young's modulus of steel & the energy stored in the wire.

☆ Soln

$\rho = 8000\,kg/m^3$, $m = 20g = 2\times10^{-2}\,kg$, $L = 2.5m$, $e = 1mm = 10^{-3}\,m$, $F = 80\,N$, $Y = ?$, $E = ?$

Now,

$\rho = \frac{M}{V}$

or, $8000 = \frac{2\times10^{-2}}{A\cdot L}$

or, $A\cdot2.5 = 2.5\times10^{-6}$

$\Rightarrow \boxed{A = 10^{-6}\,m^2}$

Now,

$Y = \frac{F}{A}\times\frac{L}{e}$

$= \frac{80\times2.5}{10^{-6}\times10^{-3}}$

$\boxed{\therefore Y = 2\times10^{11}\,N/m^2}$

Again,

$E = \frac{Fe}{2}$

$= \frac{80\times10^{-3}}{2}$

$\therefore \boxed{E = 40\times10^{-3}\,J}$ #

Thus, Youngs modulus $= 2\times10^{11}\,N/m^2$ & energy stored $= 40\times10^{-3}\,J$ #

Q. 3[C] What force is required to stretched a stell wire of cross-sectional area 1 cm^2 to double its length?

★ Soln:-

$A = 1\ cm^2 = 10^{-4}\ m^2$, $l = L$, $F = ?$, $Y = 2\times10^{11}\ N/m^2$

Now, $Y = \frac{F}{A} \times \frac{L}{l}$

or, $2\times10^{11} = \frac{F}{10^{-4}} \times \frac{L}{L}$

$\Rightarrow \boxed{F = 2\times10^{7}\ N}$ thus, required force is 2×10^{7} N. #

Q. 3[D] Calculate the work done in stretching a steel wire 100 cm in length and of cross-sectional area 0.030 cm^2 when a load of 100 N is slowly applied before the elastic limit is reached?

★ Soln:-

$L = 100cm = 1\ m$

$A = 0.030\ cm^2 = \frac{0.030}{100\times100}\ m^2 = 0.000003\ m^2$

$F = 100\ N$, $Y = 2\times10^{11}\ N/m^2$, $W = ?$

Now;

$Y = \frac{F}{A} \times \frac{L}{l}$

or, $2\times10^{11} = \frac{100\times1}{0.000003\times l}$

or, $\boxed{l = 1.67\times10^{-4}\ m}$

Again,

$W = \frac{Fl}{2}$

$= \frac{100\times1.67\times10^{-4}}{2}$

$\therefore W = 8.33\times10^{-3}$ J #

Thus, total work done $= 8.33\times10^{-3}$ J #

Q.3[E] A uniform steel wire of density 7800 Kgm^{-3} weights 16 gm and is 250 cm long. It lengthens by 1.2 mm when stretched by a force of 80 N. Calculate the Young's modulus and the energy stored in the wire.

Soln:-

$\rho = 7800\ kg/m^3$, $m = 16gm = 1.6\times10^{-2}\ kg = 0.016\ kg$, $L = 250cm = 2.5\ m$., $e = 1.2\ mm = 1.2\times10^{-3}\ m$.

$F = 80N$, $Y = ?$ & $E = ?$

Now;

$$\rho = \frac{m}{V}$$

or, $$7800 = \frac{0.016}{A\times2.5}$$

$$\Rightarrow \boxed{A = 8.2\times10^{-7}\ m^2}$$

then, $$Y = \frac{F}{A}\times\frac{L}{e} = \frac{80\times2.5}{8.2\times10^{-7}\times1.2\times10^{-3}}$$

$$\therefore Y = 2\times10^{11}\ N/m^2$$

Again,
$$E = \frac{Fe}{2} = \frac{80\times1.2\times10^{-3}}{2} = 4.8\times10^{-2}\ J$$

Thus, Young's modulus $= 2\times10^{11}\ N/m^2$ & Energy stored is $4.8\times10^{-2}\ J$. #

Q.3[F] A steel cable with cross-sectional area $3cm^2$ has an elastic limit of 2.4×10^8 pa. Find the maximum upward acceleration that can be given a 1200 Kg elevator supported by the cable if the stress is not exceed one third of elastic limit.

Soln:-

$A = 3cm^2 = 3\times10^{-4}\ m^2$

Elastic Limit $= 2.4\times10^8$ pa, stress $= \frac{1}{3}$ Elastic limit $= 8\times10^7$ pa

$M = 1200$ Kg.

Now,

$$\text{stress} = \frac{F}{A} = \frac{ma}{A}$$

or $$8\times10^7 = \frac{1200\times a}{3\times10^{-4}}$$

$$\Rightarrow \boxed{a = 20\ m/s^2}$$

Thus, required acceleration is $20\ m/s^2$. #

Q.3[G] A vertical brass rod of circular section is loaded by placing a 5 kg weight on top of it. If its length is 50 cm & radius of cross section is 1 cm, find the contraction of rod and the energy stored in it.

Soln:-

$m = 5\,kg$, $L = 50\,cm = 0.5\,m$, $r = 1\,cm = 0.01\,m$, $A = \pi r^2 = \pi(0.01)^2 = \pi \times 10^{-4}\,m^2$

$e = ?$, $E = ?$, $Y = 0.91 \times 10^{11}\,N/m^2$, $F = mg = 5 \times 10 = 50\,N$

Now;

$Y = \frac{F}{A} \times \frac{L}{e}$

or, $0.91 \times 10^{11} = \frac{50 \times 0.5}{\pi \times 10^{-4} \times e}$

$\Rightarrow \boxed{e = 8.74 \times 10^{-7}\,m}$

Again, $E = \frac{1}{2} Fe = \frac{1}{2} \cdot 50 \times 8.74 \times 10^{-7}$

$\therefore \boxed{E = 2.18 \times 10^{-5}\,J}$

Thus, Contraction of rod is $8.74 \times 10^{-7}\,m$ & stored energy is $2.18 \times 10^{-5}\,J$. #

Q.3[H] Calculate the workdone is stretching a steel wire 100 cm in length and of cross sectional area 0.03 cm² when a load of 100 N is slowly applied without the elastic limit being reached?

Soln:-

$L = 100\,cm = 1\,m$, $A = 0.03\,cm^2 = 3 \times 10^{-6}\,m^2$, $F = 100\,N$, $Y = 2 \times 10^{11}\,N/m^2$

Now,

$Y = \frac{F}{A} \times \frac{L}{e}$

or, $2 \times 10^{11} = \frac{100 \times 1}{3 \times 10^{-6} \times e}$

$\Rightarrow \boxed{e = 1.67 \times 10^{-4}\,m}$

Again, Energy Stored (E) $= \frac{Fe}{2}$

$= \frac{100 \times 1.67 \times 10^{-4}}{2}$

$\therefore E = 8.3 \times 10^{-3}\,J$ #

Q.14] How much force is required to punch a hole 1 cm in diameter in a steel sheet 5mm thick whose shearing strength is 2.76×10^{8} N/m²?

★ Soln:-

F=?, diameter of hole (d) = 1 cm = 0.01 m.

thickness of sheet (t) = 5mm = 0.005 m.

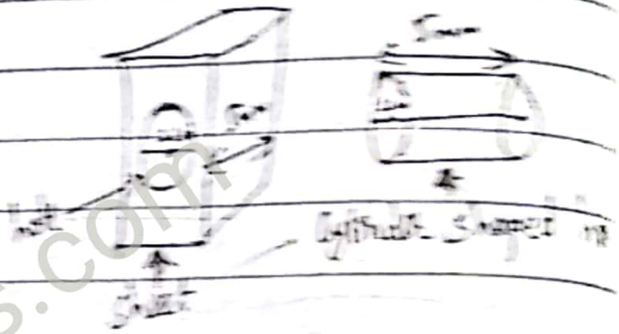
Cross-sectional area (A) = $2\pi r.t = \pi d t = \pi\times0.01\times0.005 = 0.00005\pi$ m²

Now,

Shearing Strength (η) = 2.76×10^{8} N/m²

or, $\frac{F}{A} = 2.76\times10^{8}$

or, $\frac{F}{0.00005\pi} = 2.76\times10^{8}$



$\Rightarrow \boxed{F = 4335.39\ N}$

Thus, required force is 4335.39 N

Q.15] Find the work done in stretching a wire of cross-sectional area 10^{-2} cm² and 2m long through 0.1 mm, if Y for the material of wire is 2×10^{11} N/m².

★ Soln:-

W=?, $A = 10^{-2}$ cm² $= 10^{-6}$ m², L = 2m, l = 0.1 mm = 10^{-4} m, Y = 2×10^{11} N/m²

Now,

$Y = \frac{F}{A}\times\frac{L}{l}$

or, $2\times10^{11} = \frac{F\times2}{10^{-6}\times10^{-4}}$

$\Rightarrow \boxed{F = 10\ N}$

∴ work done $= \frac{Fl}{2} = \frac{10\times10^{-4}}{2}$

∴ $W = 5\times10^{-4}$ J

Thus, work done is 5×10^{-4} J

Q.6[A] A steel wire of density 8000 $kg m^{-3}$ weights 24 g and is 250 cm long. It lengthens by 1.2 mm when stretched by a force of 80N. Calculate the Young's modulus for the steel and the energy stored in the wire.

Soln:-

$\rho = 8000\ kg/m^3$, $m = 24g = 24\times10^{-3}\ kg$, $L = 250cm = 2.5m$, $e = 1.2mm = 1.2\times10^{-3}\ m$, $F = 80N$,

We have,

$\rho = \frac{m}{V}$

or, $8000 = \frac{24\times10^{-3}}{A\times2.5}$

$\Rightarrow \boxed{A = 1.2\times10^{-6}\ m^2}$

Again, $Y = \frac{F}{A}\times\frac{L}{e} = \frac{80\times2.5}{1.2\times10^{-6}\times1.2\times10^{-3}}$

$\therefore \boxed{Y = 1.4\times10^{11}\ N/m^2}$ #

Thus, Energy stored in wire $(E) = \frac{Fe}{2} = \frac{80\times1.2\times10^{-3}}{2} = 0.048\ J$ #