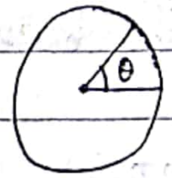
CHAPTER 06

# Circular motion:-

## # Aungular displacement:-

↳ The angle swept out (made) by position vector at the centre of circular path is called angular displacement. It is denoted by θ and it's unit is redian(rad).



## # Angular velocity(ω):-

↳ The rate of change of angular displacement is called angular velocity.

i.e,

$$\text{Angular velocity}(\omega) = \frac{d\theta}{dt}$$

And it's SI unit is rad/sec.

## # Angular acceleration:-

↳ The rate of change of angular velocity is called angular acceleration.

i.e.,

$$\text{Angular acceleration}(\alpha) = \frac{d\omega}{dt}$$

and it's SI unit is rad/sec².

## # Frequency:-

↳ The total number of complete revolution made by body in one second is called frequency.

i.e, $\text{frequency}(f) = \frac{1}{T}$

It's SI unit is Herz(Hz).

## # Time period (T):-

↳ The time taken by body to complete one revolution is called time period.

Note:  $l = \theta r$

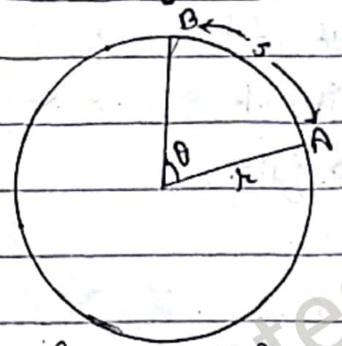


★ Since the angular displacement is $2\pi$ in one revolution,
So, angular velocity is,

$$\omega = \frac{2\pi}{T}$$

$$\therefore \omega = 2\pi f \quad \left[\because f = \frac{1}{T}\right]$$

# Relation between linear velocity and angular velocity:-






Let us Consider a body is moving in a circular path of radius 'r'. Suppose the body is initially a point A and after time 't' it reach at point B with angular displacement 'θ' & Linear displacement 's'.

Now,
From Figure;

$s = \theta r$ ----(i)

Differentiating eqn (i) w.r.t. 't';

$$\frac{ds}{dt} = \frac{d}{dt}(\theta r) = r \cdot \frac{d\theta}{dt}$$

Since, $\frac{ds}{dt} = v$ & $\frac{d\theta}{dt} = \omega$

$\therefore \boxed{v = r.\omega}$ ------(ii)

eqn (ii) gives the relation between angular velocity & linear velocity.

Again,
Differentiating eqn (ii) w.r.t. 't';

$$\frac{dv}{dt} = \frac{d}{dt}(r\omega)$$

$$\frac{dv}{dt} = r.\frac{d\omega}{dt}$$

Since, $\frac{dv}{dt} = a$ and $\frac{d\omega}{dt} = \alpha$

$\therefore \boxed{a = \alpha r}$ -----(iii)

eqn (iii) gives the relation between linear acceleration & angular acceleration.

# Expression for centripetal acceleration:-

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↳ Let us Consider a body of mass 'm' is moving in a Circular path of radius 'r' with uniform angular Velocity 'ω'. Suppose the body is initially at point A and after time 't' it reach at point p(x,y) with angular displacement. [$\theta = \omega t$]

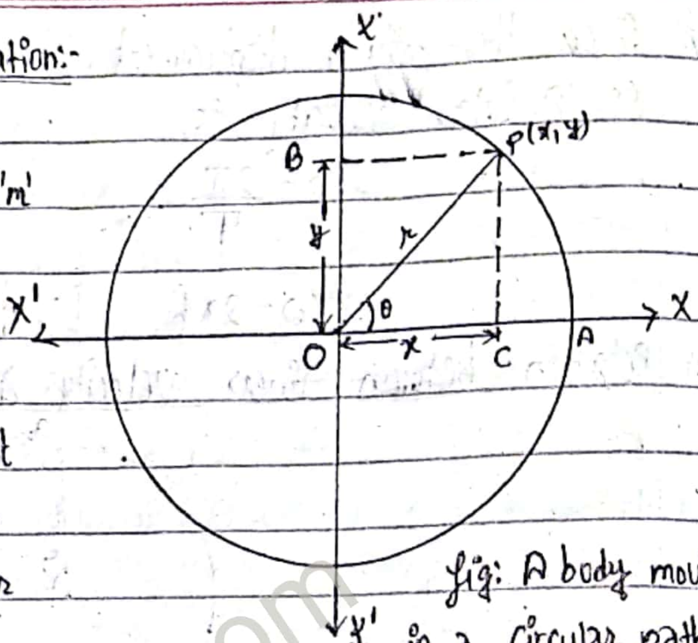



fig: A body moving in a Circular path

Now,

position vector after time 't' is given by;

$$\vec{r} = \hat{i}x + \hat{j}y$$

$$= \hat{i}(r\cos\theta) + \hat{j}(r\sin\theta)$$

$$= \hat{i}(r\cos\omega t) + \hat{j}(r\sin\omega t)$$

$$\therefore \vec{r} = r(\hat{i}\cos\omega t + \hat{j}\sin\omega t) \text{ ---- (i)}$$

Again, Velocity of a particle at time 't' is given by;

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$= \frac{d}{dt}[r(\hat{i}\cos\omega t + \hat{j}\sin\omega t)]$$

$$= r\left[\hat{i}\,\frac{d[\cos\omega t]}{d(\omega t)} \times \frac{d(\omega t)}{dt} + \hat{j}\,\frac{d(\sin\omega t)}{d(\omega t)} \times \frac{d(\omega t)}{dt}\right]$$

$$\therefore \vec{v} = \omega r \cdot [-\hat{i}\sin\omega t + \hat{j}\cos\omega t] \text{ ---- (ii)}$$

And,

Acceleration of particle at time 't' is given by;

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left[\omega r(-\hat{i}\sin\omega t + \hat{j}\cos\omega t)\right] = \omega r.\left[\frac{-\hat{i}\, d[\sin\omega t]}{d(\omega t)} \times \frac{d(\omega t)}{dt} + \hat{j}\frac{d[\cos\omega t]}{d(\omega t)} \times \frac{d(\omega t)}{dt}\right]$$

or, $\vec{a} = \omega^2 r[-\hat{i}\cos\omega t - \hat{j}\sin\omega t]$

or, $\vec{a} = -\omega^2 r(\hat{i}\cos\omega t + \hat{j}\sin\omega t)$

$\therefore \boxed{\vec{a} = -\omega^2 \vec{r}}$ --- (iii) [$\because$ using eqn(i)]

Negative sign shows that acceleration is directly towards the centre of circular path.

Hence, the magnitude of centripetal acceleration is given by;

$\boxed{a = \omega^2 r}$ ------ (iv)

This is required expression for centripetal acceleration.

# Expression for centripetal force;

↳ Centripetal force is the force required to move a body uniformly in a circle. This force act along the radius and directed towards centre of circular path.

Centripetal force = mass x centripetal acceleration.

or, $F = m \times \omega^2 r$

$F = m\left(\frac{v}{r}\right)^2 \times r$ [$\because v = \omega r$]

$\therefore \boxed{F = \frac{mv^2}{r}}$ ------ (V)

This is required expression for centripetal force.

# Motion of Cyclist in Circular path:

↳ Let us Consider a cyclist is moving in a Circular path of radius 'r' with unifor velocity 'v'. Also, Let R be the reaction and θ be the angle of inclination with vertical then reaction (R) Can be resolved into two Components. One is $R\sin\theta$ along horizontal which provides necessary centripetal force,

i.e., $R\sin\theta = \frac{mv^2}{r}$ ----- (i)

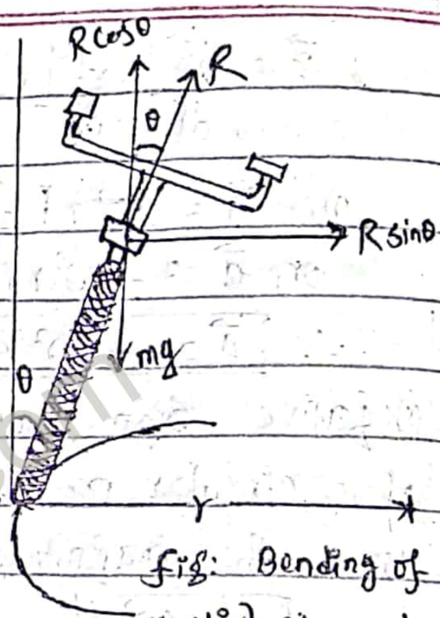



fig: Bending of a cyclist at circular path.

And Other is $R\cos\theta$ along vertical which balance the weight of Cyclist.

i.e., $R\cos\theta = mg$ ---- (ii)

Dividing eq$^n$ (i) by (ii)

or, $\frac{R\sin\theta}{R\cos\theta} = \frac{\frac{mv^2}{r}}{mg}$

$\Rightarrow \boxed{\tan\theta = \frac{v^2}{rg}}$ ----- (iii)

eq$^n$ (iii) shows that θ will be increased when v is increased and r is decreased.

# Motion of car in Curved (Circular) path:

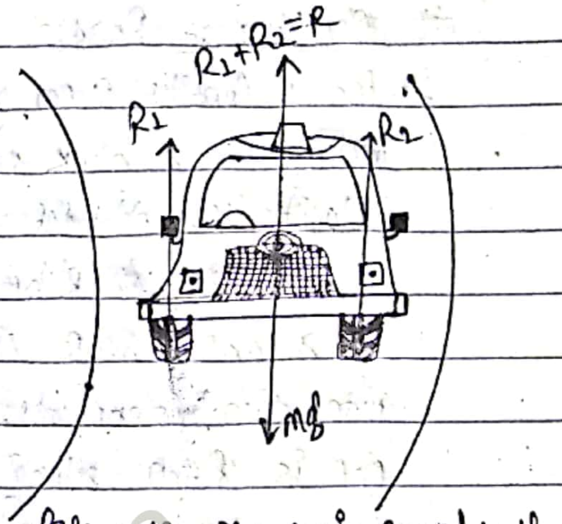


fig: Motion of a car in curved path

↳ Let us consider a car (vechile) of mass 'm' is moving in a circular path of radius 'r' with constant velocity 'v'. Also let $R_1$ & $R_2$ be the reaction at left & right side of tires.
Such that total reaction, ($R=R_1+R_2$).
Which balance the weight of a car.
i.e., $R = mg$ ----(i)
Here, the frictional force provides the necessary centripetal force.
i.e, $F_f = F_c$

or, $\mu R = \frac{mv^2}{r}$

or, $\mu mg = \frac{mv^2}{r}$ [∵ using eqn (i)]

∴ $\boxed{v = \sqrt{\mu r g}}$ --- (ii)

This is the maximum velocity with which a vechile can take a safe circular term of radius 'r'.

# Banking of Road:-

↳ Let us consider a car of mass 'm' is moving in a bank road with uniform velocity 'v'. Also let θ be the angle of banking & 'R' be the total reaction of a car. Then R can be resolved into two components.

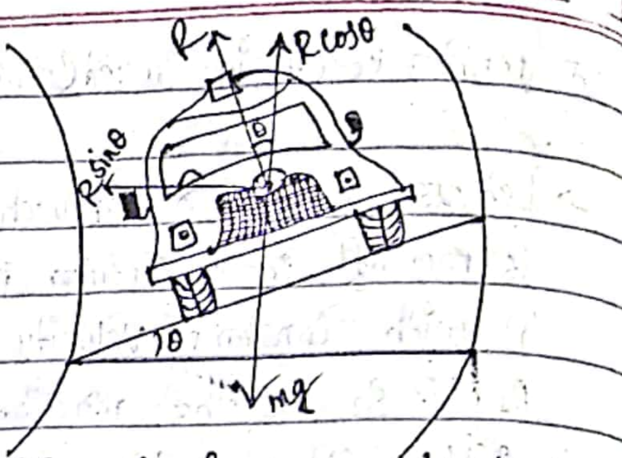



fig: Motion of a car on a banking track.

One is $R\sin\theta$ along horizontal which provides necessary centripetal force.

i.e. $R\sin\theta = \frac{mv^2}{r}$ ------(i)

And other is $R\cos\theta$ along vertical which balance the weight of a car.

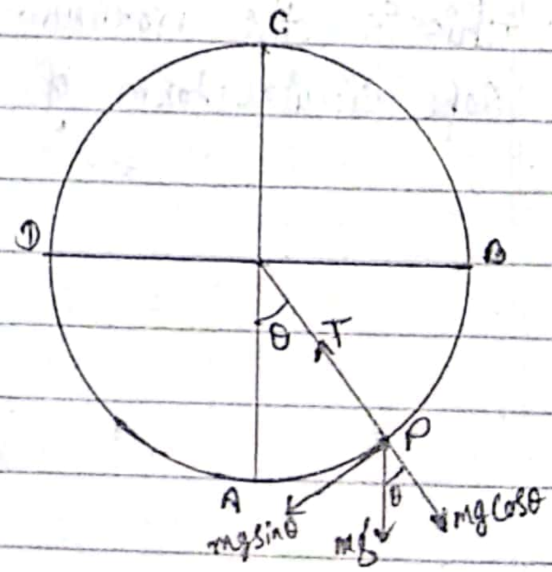
i.e., $R\cos\theta = mg$

Dividing eqn (i) by (ii), we get,

$$\boxed{\tan\theta = \frac{v^2}{rg}} \text{ ---(iii)}$$

This equation is required expression for baking of road.

# Motion in a vertical Circle:-




**AT point P:**

$F_c = T - mg\cos\theta$

$T = F_c + mg\cos\theta$

$T = \frac{mv^2}{r} + mg\cos\theta$

**AT point A | i.e, θ=0°;**

$T_{max} = \frac{mv^2}{r} + mg$

**AT point C | θ=180°**

$T_{min.} = \frac{mv^2}{r} - mg$

**AT point B & D | i.e, θ=90° & 270°**

$\Rightarrow \left\{T = \frac{mv^2}{r}\right\}$

# Numericals →

LQ. 1[A] An object of mass 8.0 kg is whirled round in a vertical circle of radius 2m with a constant speed of 6 ms⁻¹. Calculate the maximum & minimum tensions in string.

☆ Soln:-

mass(m) = 8 kg, radius of vertical circle (r) = 2m, Velocity (v) = 6 ms⁻¹

$T_{max}$ & $T_{min}$ = ?

Now,

We have, $T_{max} = \frac{mv^2}{r} + mg$ And, $T_{min} = \frac{mv^2}{r} - mg$

$= \frac{8(6)^2}{2} + 8(10)$ $= \frac{8(6)^2}{2} - 8(10)$

$\therefore \boxed{T_{max} = 224\ N}$ # $\therefore \boxed{T_{min} = 64\ N}$ #

Thus, In the string maximum tension is 224N & Minimum tension is 64 N. #

LQ. 1[B] A mass of 0.2 kg is rotated by a string at a constant speed in a vertical circle of radius 1 m. If the minimum tension in the string is 3N, calculate the magnitude of the speed & the maximum tension in the string.

☆ Soln:-

mass(m) = 0.2 kg, radius(r) = 1 m, $T_{min}$ = 3 N, Speed(v) = ?, $T_{max}$ = ?

Now,

we have, $T_{min} = \frac{mv^2}{r} - mg$

or, $3 = \frac{(0.2)v^2}{1}$

$\Rightarrow \boxed{v = 5\ ms^{-1}}$ #

Again,

We know, $T_{max} = \frac{mv^2}{r} + mg$

$= \frac{(0.2)(5)^2}{1} + (0.2)(10)$

$\therefore \boxed{T_{max} = 7\ N}$ #

Hence, speed in the string is 5 ms⁻¹ & Maximum tension is 7N. #

Q.1[C] At what angle should a circular road be banked so that a car running at 50 kmhr⁻¹ be safe to go round the circular turn of 200m radius.

☆ Soln:-

Let, angle of banking = $\theta$ = ?, Speed ($v$) = 50 kmh⁻¹ = $\frac{50\times1000}{60\times60}$ = 13.89 ms⁻¹.

radius ($r$) = 200 m.

Now, We have, $\tan\theta = \frac{v^2}{rg}$

$$\text{or, } \tan\theta = \frac{(13.89)^2}{200(10)} = \frac{192.9}{2000}$$

$$\Rightarrow \theta = \tan^{-1}[0.096]$$

$$\therefore \boxed{\theta = 5.5^\circ}$$

Thus, 5.5° angle should be banked in a circular path.



Q. 1[D] An object of mass 4.0 kg is rotated in a vertical circle of radius 1 m. with a constant speed of 3 ms⁻¹. Calculate the maximum tension in the string.

☆ Soln:- Given;

mass ($m$) = 4 kg, radius ($r$) = 1 m.

Speed of the string ($v$) = 3 ms⁻¹. Maximum tension ($T_{max}$) = ?

Now,

we have,

$$T_{max} = \frac{mv^2}{r} + mg$$

$$= \frac{4(3)^2}{1} + 4(10)$$

$$= 36 + 40$$

$$\therefore \boxed{T_{max} = 76\ N}$$

Thus, maximum tension on the string is 76 N. #

Q.1[E] A coin placed on a disc rotates with speed of $33\frac{1}{3}$ rev. min$^{-1}$ provided that the coin is not more than 10 cm. from the axis. Calculate the coefficient of static friction between the coin & the disc.

☆ Soln:-

Revolution per minute = $33\frac{1}{3}$ rev. min$^{-1}$ = $\frac{100}{3}$ rev./min

frequency per second = $\frac{100}{3} \times \frac{1}{60}$ rev./sec = $\frac{5}{9}$ rev. sec$^{-1}$

Now, radius(r) = 10 cm = 0.1 m. and

Angular velocity ($\omega$) = $\frac{v}{r}$ & $v = r.\omega$

and we have, $\omega = 2\pi f = 2\pi \times \frac{5}{9} = \frac{10\pi}{9}$ rad/sec

& $v = r.\omega = (0.1)\left(\frac{10\pi}{9}\right)$

$$\boxed{v = \frac{\pi}{9}\ ms^{-1}}$$

Again, we have, $v = \sqrt{\mu r g}$ & $\mu = \frac{v^2}{rg}$

or, $\mu = \frac{\left(\frac{\pi}{9}\right)^2}{0.1(10)} = \left(\frac{\pi}{9}\right)^2 = \left(\frac{22}{7} \times \frac{1}{9}\right)^2 = 0.121945$ #

Thus, coefficient of static friction between coin & disc is 0.122 #

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# # Conical pendulum (Horizontal pendulum)

↳ A system consisting of a small/heavy bob suspended by a string from a rigid support and whirled round in a horizontal circle at a constant speed is called conical pendulum.

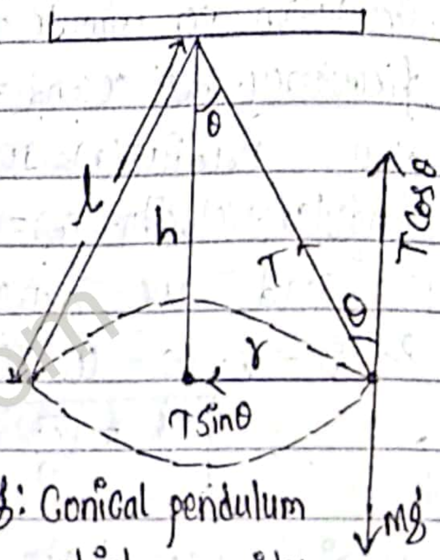



fig: Conical pendulum

★ Let us consider a small bob of mass 'm' is suspendedly a string of length 'l' from a regid support. Now the bob is whirled in a horizontal circle of radius 'r' with constant velocity 'v'. Such that at any point the string inclined by angle 'θ' with vertical height 'h'. Then the tension(T) can be resolved into two components. One is $T\sin\theta$ which provides necessary centripetal force.

i.e., $T\sin\theta = \frac{mv^2}{r}$ ---- (i)

And other is $T\cos\theta$ which balances weight of the bob.

i.e, $T\cos\theta = mg$ ---- (ii)

dividing eq$^n$(i) by (ii), we get;

$\tan\theta = \frac{v^2}{rg}$ ---- (iii)

Also, from figure;

$\tan\theta = r/h$ ---- (iv)

From eq$^n$ (iii) & eq$^n$ (iv)

$\frac{r}{h} = \frac{v^2}{rg}$

⇒ $\frac{r}{v} = \sqrt{\frac{h}{g}}$ ---- (v)

If 't' be the time period of conical pendulum then, $t = \frac{2\pi r}{v}$ ---- (vi) $[\because t = \frac{\text{distance}}{\text{velocity}}]$

Using eq$^n$ (v) in eq$^n$ (vi)

or, $t = 2\pi\sqrt{h/g}$ ---- (vii)

Also, from figure,

$\cos\theta = h/l$

$h = l\cos\theta$ ---- (viii)

Using eq$^n$ (viii) in eq$^n$ (vii)

$\therefore \boxed{t = 2\pi\sqrt{\frac{l\cos\theta}{g}}}$ ---- (ix)

Q.2[A] A bob of mass 200 gram is whirled in a horizontal circle of radius 50 cm by a string inclined at 30° to the vertical. Calculate the tension in the string & the speed of the bob in the horizontal circle.

★ Soln:-

$m = 200 \text{ gram} = 0.2 \text{ kg}, \; r = 50 \text{ cm} = 0.5 \text{ m}., \; \theta = 30°, \; T = ?, \; v = ?$

Now, $T\cos\theta = mg$ and, $T\sin\theta = \frac{mv^2}{r}$

or, $T \cos 30° = (0.2)(10)$ or, $2.3 . \sin 30° = \frac{(0.2)(v^2)}{0.5}$

$\Rightarrow \boxed{T = 2.3}$ N #

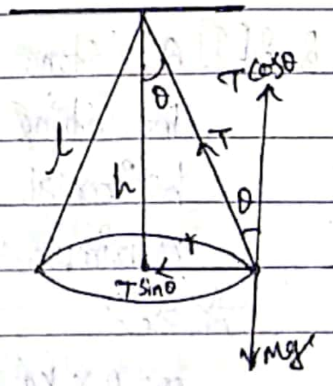
$\Rightarrow \boxed{v = 1.699 \text{ ms}^{-1}}$ #

Thus, tension in the string is 2.3 N & speed of the bob is 1.699 ms⁻¹! #



Q.2[B] An object of mass 0.5 kg is rotated in a horizontal circle by a string in 1 m long. The maximum tension in the string before it brakes is 50 N. What is the greatest number of revolutions per second of the object?

★ Soln:-




$m = 0.5 \text{ kg}, \; l = 1 \text{ m}., \; T = 50 \text{ N}.,$

numbers of revolutions per second $= ? = f$

We have, $T\cos\theta = mg$

or, $50 \cdot \cos\theta = (0.5)(10)$

$\Rightarrow \cos\theta = \frac{5}{50} = 0.1$

and, time period,

$t = 2\pi\sqrt{\frac{l\cos\theta}{g}} = 2(3.14) \times \sqrt{\frac{1 \cdot (0.1)}{10}} = (6.28)(\sqrt{0.01}) = 0.628$

We know, $f = \frac{1}{t} = \frac{1}{0.628} = 1.59$ rev/sec.

Thus, the greatest numbers of revolutions per second of the object is 1.6 rev/sec. #

Q.2 [C] A Certain String breaks when a weight of 25 N acts on it. A mass of 500 gram is attached to one end of the String of 1 m long and is rotated in a horizontal Circle. Find the greatest number of revolutions per minute which Can be made without breaking the String.

Soln:- Given,

$T = 25$ N., $m = 500$ gram $= 0.5$ kg., $l = 1$ m,

revolution per minute $= ?$

Now; we have,

$T\cos\theta = mg$

or, $25\cos\theta = 0.5(10)$

$\Rightarrow \boxed{\cos\theta = \frac{5}{25} = \frac{1}{5}}$

And, $t = 2\pi\sqrt{\frac{l\cos\theta}{g}}$

$= \frac{2\times22}{7}\sqrt{\frac{1(1/5)}{10}}$

$\Rightarrow \boxed{t = 0.8889 \text{ sec.}}$

Again,

frequency per second $= \frac{1}{t} = \frac{1}{0.8889} = 1.1249$ rev/sec

& Revolution per minute $= 1.1249 \times 60 = 67.49$ rev/min #

Q.2 [D] A Stone with mass 0.8 kg is attached to one end of a String 0.9 m long. The String will break if its tension exceeds 600 N. The stone is whirled in a horizontal Circle, the other end of the String remains fixed. Find the maximum Speed, the Stone can attain without breaking the String.

Soln:-

$m = 0.8$ kg, $l = 0.9$ m., $T = 600$ N., $v_{max} = ?$, here, $l = r = 0.9$ m.

Now,

we have, $T\cos\theta = mg$

or, $600\cos\theta = 0.8(10)$

$\Rightarrow \boxed{\theta = \cos^{-1}\left(\frac{8}{600}\right) = 89.23^\circ}$

Also, we know,

$T\sin\theta = \frac{mv^2}{r}$

or, $600\sin(89.23)^\circ = \frac{0.8(v^2)}{0.9}$

$\Rightarrow \boxed{v = 25.98 \text{ ms}^{-1}}$ #

Hence, maximum Speed is $25.98$ ms$^{-1}$. #

Q.2[6] A mass of 1 kg is attached to the lower end of a string 1 m long whose upper end is fixed. The mass is made to rotate in a horizontal circle of radius 60 cm. If the circular speed of the mass is constant, find the tension in the string & the period of motion.

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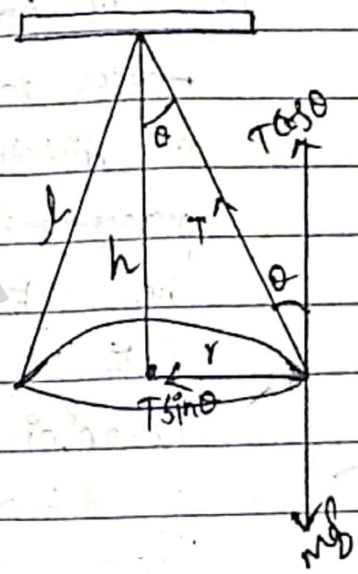
★ Soln:-




$m = 1\ kg$, $l = 1m$, $r = 60cm = 0.6m$, $t = ?$, $T = ?$

Now,

From figure, $\sin\theta = \frac{r}{l} = \frac{0.6}{1}$

$\Rightarrow \boxed{\theta = 36.86^\circ}$

Also, we have,

$T\cos\theta = mg$

or, $T = \frac{1(10)}{\cos(36.86)^\circ} = \frac{10}{0.8} = 12.5\ N$

and,

$t = 2\pi\sqrt{\frac{l\cos\theta}{g}} = 2\times\frac{22}{7}\times\sqrt{\frac{1(0.8)}{10}} = 1.77\ sec$

Hence, tension (T) on the string is 12.5 N & time period is 1.77 sec. #