KINEMATICS

Static:-

The branch of mechanics which deals with body at rest is called static.

Kinematics:-

The branch of mechanics which deals with the study of the motion of objects without taking into the account of the cause of the motion in the objects.

Dynamics:-

The branch of mechanics which deals with the study of the motion of the objects with talking into the account of the cause of the motion in the objects.

Displacement and distane:-

The shortest distance between two point is called displacement. It is a vector quantity directed along the direction of motion.

The length of actual path travelled by body between two point is called distance. It is a scalar quantity.

Q.) Can a body have zero displacement but non-zero distance travelled?

★ Soln Yes, it can be, a body returns to same position after motion, then displacement is zero but distance travelled is non-zero.

Speed and Velocity:-

The rate of change of displacement of a body is called velocity.

i.e, $velocity = \frac{displacement}{time\ taken}$

It is a vector quantity and it's SI unit is m/s.

The rate of change distance travelled by a body is called speed.

i.e, $speed = \frac{distane}{time\ taken}$

It is a scalar quantity and it's SI unit is m/s.

Average Speed and instantaneous speed:

Average speed of a body is defined as the ratio of total distance travelled to the total time taken.

i.e, $Average\ Speed = \frac{Total\ distane\ travelled}{total\ time\ taken}$

The speed of a body at a particular instant of time on it's path is called instantaneous speed.

Average velocity and instantaneous Velocity:-

Average velocity of a body is defined as the ratio of total displacement to the total time taken.

i.e, $\text{Average velocity} = \dfrac{\text{Total displacement}}{\text{Total time taken}}$

The velocity of a body at a particular instant of time is called instantaneous velocity.

Acceleration or Retardation:-

The rate of change of velocity is called acceleration;

i.e, $a = \dfrac{\text{Change in velocity}}{\text{time}}$

It is a vector quantity and it's SI unit is ms^{-2}.

The rate of decrease of velocity is called retardation ~~of~~ or deacceleration or negative acceleration

$+g \downarrow \quad \uparrow -g$

Equation of motion with uniform acceleration:-

1) $V = u + at$
2) $S = ut + \frac{1}{2}at^2$
3) $V^2 = u^2 + 2as$
4) $S_{nth} = u + a\left(\frac{2n-1}{2}\right)$

Equation of motion under gravity:-

1) $V = u \pm gt$
2) $h = ut \pm \frac{1}{2}gt^2$
3) $V^2 = u^2 \pm 2gh$
4) $S_{nth} = u \pm g\left(\frac{2n-1}{2}\right)$

Distance travelled in n^{th} second:-

We have;

$$S_n = un + \frac{1}{2}an^2$$

$$S_{n-1} = u(n-1) + \frac{1}{2}a(n-1)^2$$

Now, $S_{nth} = S_n - S_{n-1}$

$$= un + \frac{1}{2}an^2 - u(n-1) - \frac{1}{2}a(n-1)^2$$

$$= un + \frac{1}{2}an^2 - un + u - \frac{1}{2}a(n-1)^2$$

$$= u + \frac{1}{2}a\left[n^2 - (n-1)^2\right]$$

$$= u + \frac{1}{2}a\left[n^2 - n^2 + 2n - 1\right]$$

$$= u + a\left(\frac{2n-1}{2}\right)$$

$$\therefore \boxed{S_{nth} = u + a\left(\frac{2n-1}{2}\right)}$$

Ans

Equation of motion (Graphical treatment)

Let us consider a body is moving with constant acceleration 'a'. Along a straight line AB with intial velocity 'u' (t=0) and after time 't' it's final velocity become 'v'.

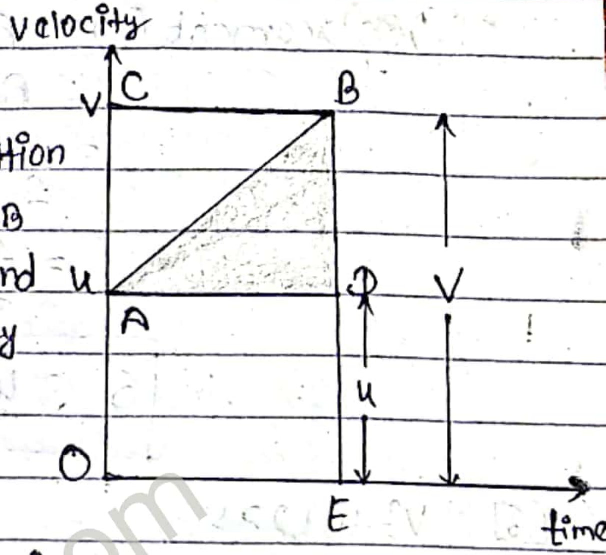


fig: velocity-time (v-t) graph with uniform acceleration

From figure;

$OA = ED = u$

$EB = OC = v$

$AD = OE = t$

[1] $v = u + at$

☆ Now, acceleration of body = Slope of line AB

$$\text{or, } a = \frac{BD}{AD} = \frac{EB - ED}{AD}$$

$$\text{or, } a = \frac{v-u}{t}$$

$$\text{or, } v - u = at$$

$$\therefore \boxed{v = u + at}$$

[2] $S = ut + \frac{1}{2}at^2$

☆ We have,

$$a = \frac{BD}{AD} = \frac{BD}{t}$$

$$\therefore BD = at$$

Displacement (S) = Area of trapezium of OABE

$= \text{Area of } \Delta ADB + \text{Area of rectangle } OADE$

$= \frac{1}{2}(BD \times AD) + AD \times ED$

$= \frac{1}{2}(at \times t) + tu$

$$\therefore \boxed{S = ut + \frac{1}{2}at^2}$$

[3] $V^2 = u^2 + 2as$

We have,

$a = \frac{BD}{AD} = \frac{EB - ED}{AD} = \frac{EB - ED}{AB}$

or, $AD = \frac{EB - ED}{AB}$

Now; Displacement (S) = Area of trapezium of OABE

or, $S = \frac{1}{2}(EB + OA) \cdot AD$

or, $S = \frac{1}{2}(EB + OA) \times \frac{EB - ED}{AB}$

or, $S = \frac{1}{2}(v + u) \frac{(v - u)}{a}$

or, $S = \frac{1}{2a}(v^2 - u^2)$

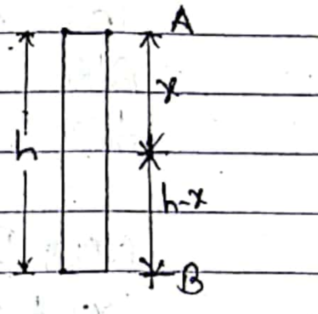
or, $v^2 - u^2 = 2as$

$$\boxed{\therefore V^2 = u^2 + 2as}$$

←Numericals⟶

Q.1[A] An object is dropped from the top of the tower of height 156.8 m and at the same time another object is thrown vertically upward with the velocity of 78.1 m/s from the foot of the tower, when and where the object meet?

★ Let A be the top of tower and B be it's foot. Also let C be the point where the ~~both~~ object meet.



Now, For dropped object,
distane travelled,

$$s = ut + \frac{1}{2}gt^2$$

$$\text{or, } x = \frac{1}{2}gt^2 \quad \text{--- (i)}$$

For object thrown vertically upward,
distane travel,

$$s = ut + \frac{1}{2}gt^2$$

$$\text{or, } h - x = 78.1t - \frac{1}{2}gt^2 \quad \text{--- (ii)}$$

using eqn (i) in eqn (ii):

$$\text{or, } h - \frac{1}{2}gt^2 = 78.1t - \frac{1}{2}gt^2$$

$$\text{or, } h = 78.1t$$

$$\text{or, } t = \frac{156.8}{78.1} = 2 \text{ sec}$$

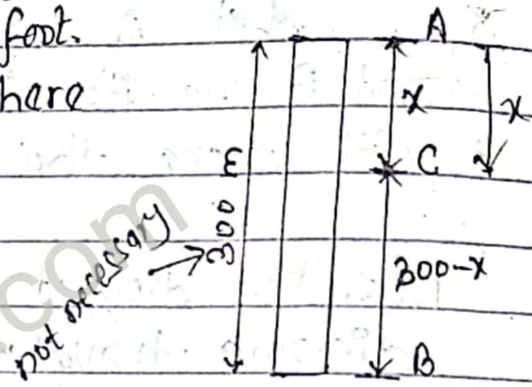
putting t=2 sec in eqn (i) we get,

$$x = \frac{1}{2} 10 \times (2)^2 = 20 \text{ m}$$

Hence, they meet 20 m below from the top of tower after 2 sec.

Q.1[B] A ball is dropped from the top of a tower 300 m height after 1 sec another ball is dropped with 20 m/s from the top of the tower. When and where they meet?

☆ Let A be the top of the tower and B be the foot. Also, Let C be the point where the both object meet:



Now; $\boxed{\text{case I}}$

$$x = \frac{1}{2}gt^2 \quad \text{—— (i)}$$

Then, $\boxed{\text{case II}}$

$$x = 20(t-1) + \frac{1}{2}g(t-1)^2 \quad \text{—— (ii)}$$

From eq^n (i) & (ii)

or, $\frac{1}{2}gt^2 = 20(t-1) + \frac{1}{2}g(t-1)^2$

or, $5t^2 = 20t - 20 + 5(t^2 - 2t + 1)$

or, $5t^2 = 20t - 20 + 5t^2 - 10t + 5$

or, $15 = 10t$

$\Rightarrow$ $t = 1.5$ sec.

putting the value of 't' in eq^n (i), we get;

or, $x = \frac{1}{2} \times 10 \times (1.5)^2$

$= 5(2.25)$

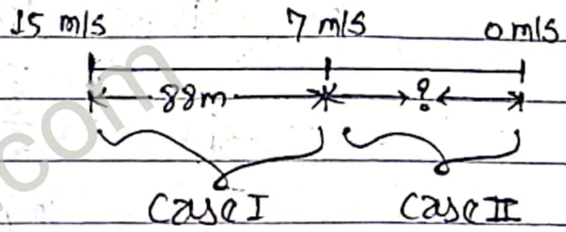
$\therefore \boxed{x = 11.25 \text{ m}}$

Hence they meet 11.25 m below from the top of tower after 1.5 seconds.

Q.1[c] A car travelling with a speed of 15 m/s is braked and it slows down with uniform retardation. It covers a distance of 88 m and it's velocity reduces to 7 m/s. If the car continuous to slow down with the same rate, after what further distance will it be broughted to rest?

☆

By the figure;



In $\boxed{\text{case I:}}$

$u = 15$ m/s, $s = 88$ m, $v = 7$ m/s

We have,

$$v^2 = u^2 + 2as$$

or, $7^2 = (15)^2 + 2 \times a \times 88$

or, $49 - 225 = 176 \cdot a$

or, $a = -1$

$\therefore$ Retardation $= 1$ m/s^2

Again; $\boxed{\text{case II:}}$

$u = 7$ m/s, $s = ?$, $v = 0$ m/s, $a = -1$ m/s^2

We have, $v^2 = u^2 + 2as$

or, $0^2 = 7^2 + 2(-1) \times s$

or, $2s = 49$

$\therefore s = 24.5$ m

Hence a car will be broughted to rest after further distance 24.5 m.

LQ.[1D] A balls falls freely from top of the tower and during the last second of it's fall. It falls through 25 m. Find the height of tower? (Ans: 45 m)

Solution:-

In last second it falls through $(S_{t^{th}}) = 25$ m.

Time taken (t) = ? sec.

initial velocity (u) = 0 ms^{-1}

Now, Distance travelled in t^{th} second;

$$S_{t^{th}} = u + \frac{g}{2}(2t-1)$$

$$\text{or, } 25 = 0 + \frac{10}{2}(2t-1)$$

$$\text{or, } \frac{25}{5} = 2t-1$$

$$\text{or, } 6 = 2t$$

$$\Rightarrow t = 3 \text{ sec.}$$

Again;

$$h = ut + \frac{1}{2}gt^2$$

$$= 0\times3 + \frac{1}{2}\times10(3)^2$$

$$= 5(9)$$

$$\Rightarrow h = 45 \text{ m}$$

Hence, Height of the tower is 45 m #

SQ.[1E] If the displacement of body is proportional to the square of time. State the nature of the motion of the body?

By question; $y \propto t^2$

$$\text{or, } y = kt^2$$

Now,

$$\text{Velocity} = \frac{dy}{dt} = \frac{d(kt^2)}{dt} = k\frac{dt^2}{dt} = 2kt$$

then,

$$a = \frac{dv}{dt} = \frac{d(2kt)}{dt} = 2k\frac{d(t)}{dt} = 2k$$

$$\Rightarrow \boxed{a = 2k}$$

Hence, the body moves with constant acceleration.

projectile motion:

Any body thrown towards space so that it moves only under the action of gravity is called projectile motion.

e.g. 1) Stone thrown horizontally
2) A bomb drop from aeroplane.

[1] projectile Fired at an angle with horizontal:-

(i) $u_x = u \cos\theta$ along horizontal and
(ii) $u_y = u \sin\theta$ along vertical.

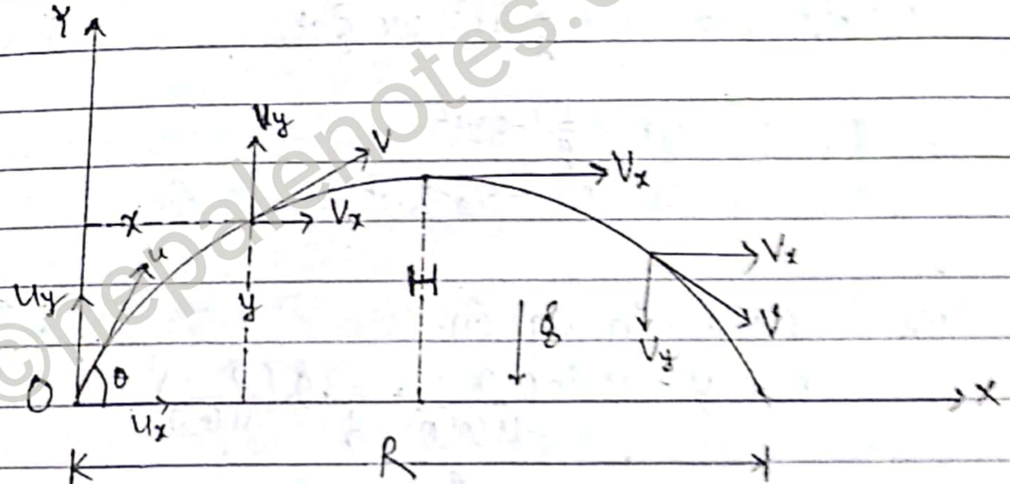


fig: projectile thrown at certain angle with horizontal

Let us consider a projectile thrown towards sky from ground with initial velocity 'u' making angle 'θ' with ground. Now it velocity can be resolved into two components. $u\cos\theta$ along horizontal and $u\sin\theta$ along vertical. Since acceleration due to gravity act in vertical direction only. So, horizontal velocity remains constant but vertical velocity is variable.

Since $p(x,y)$ be any point at which projectile takes time 't' to reach from the ground.

☆ Motion along horizontal,

Using; $S = ut + \frac{1}{2}gt^2$, we get;

or, $x = u_x t$

$= u \cos\theta . t$

$\therefore t = \frac{x}{u\cos\theta}$ ------ (i)

☆ Motion along vertical,

Using; $S = ut + \frac{1}{2}at^2$, we get,

or, $y = u_y t + \frac{1}{2}(-g)t^2$

or, $y = u\sin\theta . t - \frac{1}{2} . g . t^2$ ------ (ii)

Using eqⁿ (i) in eqⁿ (ii)

or, $y = u\sin\theta . \frac{x}{u\cos\theta} - \frac{1}{2} . g\left(\frac{x}{u\cos\theta}\right)^2$

Or, $y = \tan x - \frac{g}{2u^2\cos^2\theta} . x^2$ ------ (iii)

Which is the eqⁿ of the form $y = ax + bx^2$ with $a = \tan\theta$ & $b = \frac{-g}{2u^2\cos^2\theta}$ eqⁿ of parabola.

Hence, the path of projectile is parabolic.

★ Time of flight (T):

The time for which projectile remains in space is called time of flight.

Since, the projectile returns to ground after time T.
Now,

$$h = u_y T - \frac{1}{2} g T^2$$

or, $0 = u\sin\theta\, T - \frac{1}{2} g T^2$

or, $\frac{1}{2} g T^2 = u \sin\theta\, T$

$$\therefore \boxed{T = \frac{2u\sin\theta}{g}}$$

This is required eqⁿ for time of flight.

★ Maximum Height (H_{max}):

It is the greatest height to which a projectile rise above the point of projection.

At maximum height the vertical velocity of the projectile becomes zero. i.e, $V_y = 0$. So,

$$V_y^2 = u_y^2 - 2g H_{max}$$

or, $0 = (u\sin\theta)^2 - 2g H_{max}$

$$\therefore \boxed{H_{max} = \frac{u^2\sin^2\theta}{2g}}$$

This is expression for maximum height.

★ **Horizontal Range (R):**

The horizontal distance covered by the projectile during its time of flight is called horizontal range.

Since there is no acceleration due to gravity in horizontal range or direction.

So,

R = Horizontal velocity × time of flight

or, $R = u_x \cdot T$

or, $R = u\cos\theta \cdot \frac{2u\sin\theta}{g}$

$\therefore \boxed{R = \frac{u^2 \sin 2\theta}{g}}$ This is a expression for horizontal range.

★ **Maximum horizontal range (R_{max}):**

If, $\sin 2\theta = 1$

or, $\sin 2\theta = \sin 90°$

$\Rightarrow \theta = 45°$

Thus, the horizontal range will be maximum if the projectile is fired at an angle of 45° with the horizontal.

★ **Two angle of projectile for same horizontal range.**

We know that for the angle of projection θ, the horizontal range is given by;

$R = \frac{u^2 \sin 2\theta}{g}$

Let, R' be the horizontal range when the angle of projection is $90°-\theta$. then

$$R' = \frac{u^2 \sin 2(90°-\theta)}{g}$$

or, $$R' = \frac{u^2 \sin(180°-2\theta)}{g}$$

or, $$R' = \frac{u^2 \sin 2\theta}{g}$$

$$\boxed{\therefore R' = R}$$

Thus, we can see that for two angle of projection that is θ and $90°-\theta$ the horizontal range are same.

☆ Velocity of any instant:-

Horizontal Velocity at any instant,

$$V_x = u_x = u\cos\theta$$ and,

Vertical velocity at any instant,

$$V_y = u_y - gt$$
$$= u\sin\theta - gt$$

$\therefore$ Velocity at any instant

$$\boxed{V = \sqrt{V_x^2 + V_y^2}}$$

If θ be the angle made by Velocity with horizontal Velocity, then,

$$\tan\theta = \frac{V_y}{V_x}$$

$$\boxed{\therefore \theta = \tan^{-1}\left(\frac{V_y}{V_x}\right)}$$

Numericals →

Short Q.2[A] A projectile fired at an angle 18° has certain horizontal range. State another angle of projection for the same horizontal range?

★ There are two angle of projection for the horizontal range.

When, angle of projection is ($\theta = 18°$)

then,
$$R = \frac{u^2 \sin 2.18°}{g} = \frac{u^2 \sin 36°}{g}$$

Now,

when angle of projection is $(90°-\theta) = (90°-18°) = 72°$

then,
$$R' = \frac{u^2 \sin 2.(90°-18°)}{g} = \frac{u^2 \sin(180°-36°)}{g} = \frac{u^2 \sin 36°}{g}$$

Hence, another angle of projection is 72° for the same horizontal range.

Short Q.2[B] What to be the effect on R_{max} in doubling the intial velocity of a projectile?

★ We have,
$$R_{max} = \frac{u^2 \sin 2(45°)}{g} = \frac{u^2}{g}$$

If 'u' or intial velocity is double, $u' = 2u$

or,
$$R'_{max} = \frac{(u')^2}{g} = \frac{(2u)^2}{g} = \frac{4u^2}{g} = 4\left(\frac{u^2}{g}\right)$$

So, when doubling the intial velocity then R_{max} is increase by 4 times with intial velocity.

SQ.2[c] Find the angle of projection at which the horizontal range and Hmax of a projectile are equal?

☆

Given, $R = H_{max}$

or, $\frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin^2\theta}{2g}$

or, $\sin 2\theta = \frac{\sin^2\theta}{2}$

or, $\frac{2\sin\theta\cos\theta}{\sin\theta} = \frac{\sin\theta}{2}$

or, $4\cos\theta = \sin\theta$

or, $\frac{\sin\theta}{\cos\theta} = 4$

or, $\tan\theta = 4$

or, $\theta = \tan^{-1}(4)$

$\therefore \boxed{\theta = 75.96°}$

Hence, If R and Hmax is equal then angle of projection is 75.96°. #

LongQ.2[b] A batter hits a baseball so that it leaves the bat with an intial speed 37 ms⁻¹ at an angle of 53°. Find the position of the ball and direction of it's velocity after 2 seconds. Treat the base ball as projectile?

☆ intial velocity $(u) = 37\ ms^{-1}$

angle of projection $(\theta) = 53°$, time $(t) = 2$ sec.

Now,

We have,

$x = u_x.t = u\cos\theta.t = 37 \times \cos 53^\circ . 2 = 44.53$ m

and,

$y = \tan\theta\, x - \frac{g}{2u^2\cos^2\theta} . x^2$

$= \tan 53^\circ (44.53) - \frac{10}{2 \times (37)^2 \times (\cos 53^\circ)^2} \times (44.53)^2$

$= 59.09 - 19.99$

$= 39.1$ m

Thus, position of the ball is an point $p(44.53, 39.1)$

for the direction;

$V_x = u_x = u\cos\theta = 37 \times \cos 53^\circ = 22.26$ and

$V_y = u\sin\theta - gt$

$= 37 \times \sin 53^\circ - 10(2)$

$= 9.549$

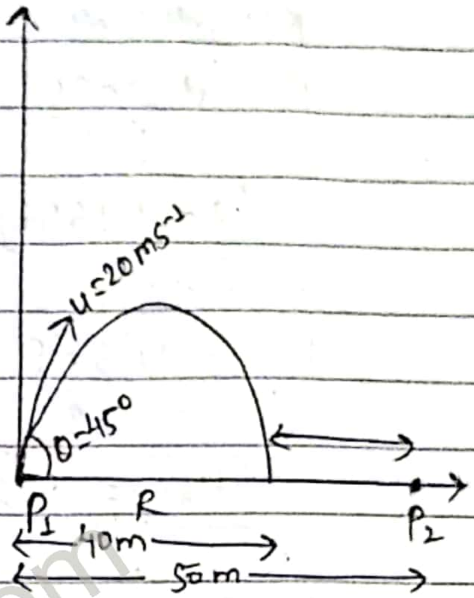
Now,

$\theta = \tan^{-1}\left[\frac{V_y}{V_x}\right] = \tan^{-1}\left(\frac{9.549}{22.26}\right) = 23.21^\circ$

So, direction is $\theta = 23.21^\circ$ #

LQ.2[E] A baseball is thrown towards a player with an intial velocity 20 ms^{-1} and 45° with the horizontal at the moment the ball is thrown, the player is 50 m from the thrower. At what speed and direction must he run to catch the ball at the same height at which it was released?

☆ Given; intial velocity $(u) = 20\ ms^{-1}$
angle of projection $(\theta) = 45°$
Now,
$$R = \frac{u^2 \sin 2\theta}{g} = \frac{(20)^2 \times \sin 2(45°)}{10} = 40\ m$$
Similarly;
$$T = \frac{2u\sin\theta}{g} = \frac{2 \times 20 \times \sin 45°}{10} = \frac{4 \times 1}{\sqrt{2}} = 2\sqrt{2}\ sec.$$



Again,
$$\text{Speed} = \frac{\text{distance}}{\text{time}} = \frac{(50-40)\,m}{2\sqrt{2}\ s} = \frac{10}{2\sqrt{2}} = \frac{5}{\sqrt{2}} = 3.53\ m/s$$

$\therefore$ speed $= 3.53\ ms^{-1}$

Hence, Second player must run towards first player with speed $3.53\ ms^{-1}$.

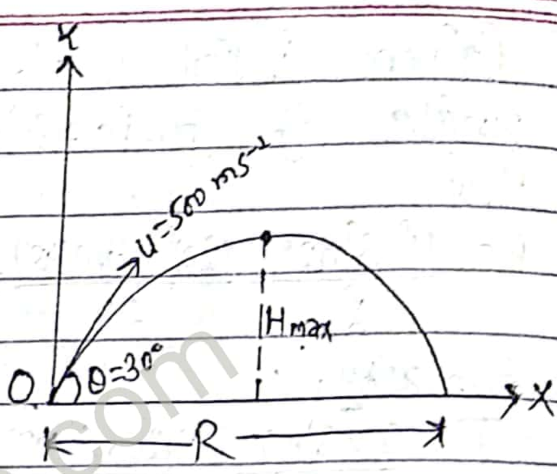
LQ.2[F] A projectile is fired from the ground level with a velocity of $500\ ms^{-1}$ at $30°$ to the horizontal. Find the horizontal range and greatest vertical height to which it rise. What is the least speed with which it could be projected in order to acheive the same horizontal range? $(g = 10\ ms^{-2})$

☆ Given;
intial velocity $(u) = 500\ ms^{-1}$
angle of projection $(\theta) = 30°$
$g = 10\ ms^{-2}$

Now, we have,

$R = \frac{u^2 \sin 2\theta}{g} = \frac{(500)^2 \times \sin 2(30°)}{10}$

$\therefore \boxed{R = 21651}$ m



Similarly;

$H_{max} = \frac{u^2 \sin^2\theta}{2g} = \frac{(500)^2 \times \sin^2 30°}{2 \times 10}$

$\therefore \boxed{H_{max} = 3125 \text{ m}}$

Again, in the case of least speed;

by using

$R = \frac{u^2 \sin 2\theta}{g}$, we got,

$R = \frac{u_{least}^2 \sin 2\theta}{g} = \frac{u_{least}^2 \cdot 1}{g}$

[Here 'u' will be least when $\because \sin 2\theta$ will be maximum. The maximum value of $\sin 2\theta = 1$.]

Or, $u_{least}^2 = Rg$

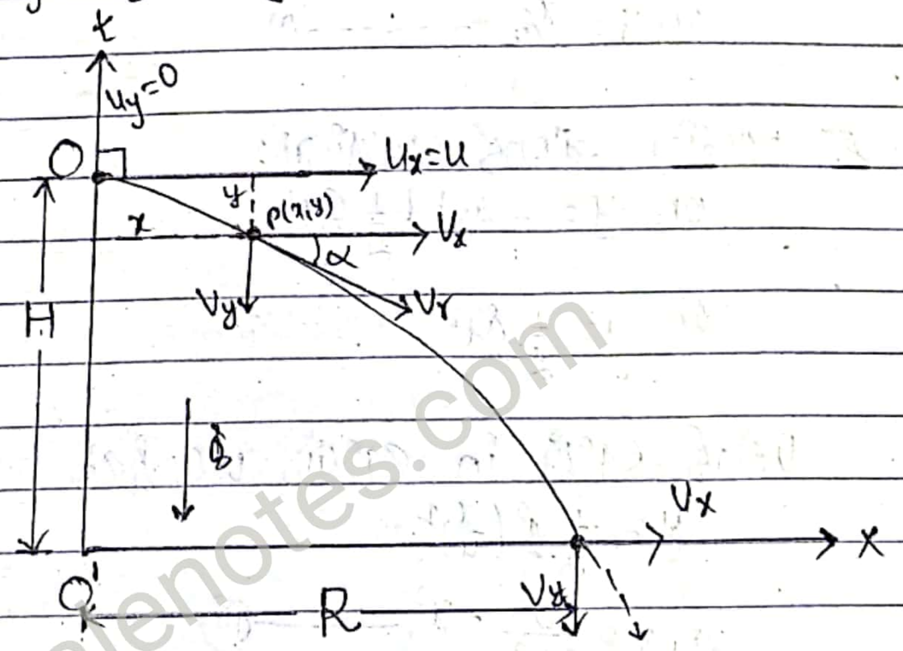
or, $u_{least} = \sqrt{Rg} = \sqrt{21651 \times 10} = \sqrt{216510}$

$\therefore u_{least} = 465.30 \text{ ms}^{-1}$

Thus, least speed is 465.30 ms⁻¹ in same horizontal range.

Horizontal projectile:

Any body thrown horizontally from the top of the tower and which moves only under the action of gravity is called horizontal projectile.



Let us consider a body is thrown horizontally from the top of tower of height 'h' and with velocity 'u'. Suppose after time 't' it reach at point $p(x,y)$ at depth 'y' and horizontal distane x from point of projection.

Since it is thrown horizontally so it's intial vertical velocity $u_y=0$ and acceleration due to gravity acts in vertical direction only. So, it's horizontal velocity remains constant.

☆ Motion along horizontal:

or, $x = u_x t$

or, $x = ut$

or, $t = x/u$ ------ (i)

☆ Motion along vertical:

or, $y = -u_y t + \frac{1}{2} g t^2$

or, $y = \frac{1}{2} g t^2$ ------ (ii)

Using eqⁿ (i) in eqⁿ (ii), we get,

$y = \frac{1}{2} g \left(\frac{x}{u}\right)^2$

or, $y = \frac{g}{2u^2} \cdot x^2$ ——— (iii)

Which is the eqⁿ of the form $y = bx^2$ with $b = \frac{g}{2u^2}$ and it is equation of parabola.
So, path of horizontal projectile is also parabolic.

☆ Time of flight (T):

The time for which projectile remains in space is called time for flight.
If 'T' be the time of flight then,

$h = -u_y T + \frac{1}{2} g T^2$

or, $h = \frac{1}{2} g T^2$

$\therefore \boxed{T = \sqrt{\frac{2h}{g}}}$ This is expression for time of flight.

☆ Horizontal Range(R):

the horizontal distance Covered by the projectile during it's time of flight is Called horizontal range.

Now,

$R =$ Horizontal velocity $\times$ time of flight

$= u_x . T$

$\therefore \boxed{R = u\sqrt{\frac{2h}{g}}}$ This is expression for horizontal range.

☆ special cases (I and II):

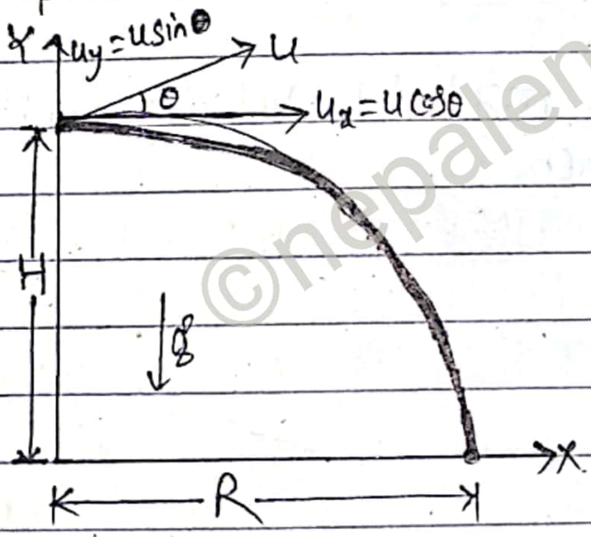


fig: case I

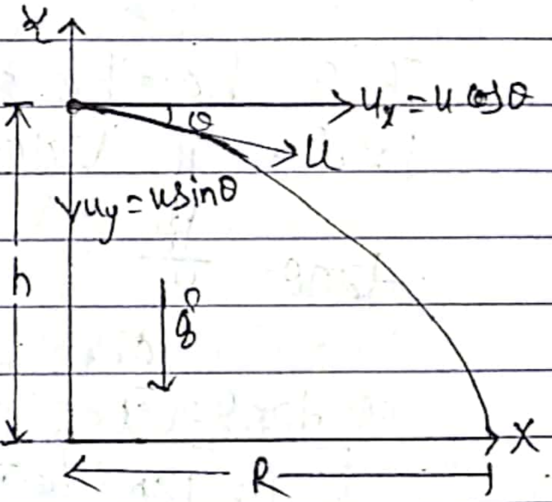


fig: case II

Case I

☆ Time of flight(T):

$h = -u_y T + \frac{1}{2} g T^2$

$h = -u\sin\theta . T + \frac{1}{2} g T^2, (T = ?)$

☆ Horizontal Range(R):

$R = u_x T$

$= u\cos\theta T$

Case II

☆ Time of flight(T):

or, $h = u_y T + \frac{1}{2} g T^2$

or, $h = u\sin\theta T + \frac{1}{2} g T^2 \ (T = ?)$

☆ Horizontal Range(R):

$R = u_x T$

$= u\cos\theta T$

★ Velocity of any instant:

Horizontal velocity at any instant;

$\therefore V_x = u_x = u$

Vertical velocity at any instant;

$V_y = u_y + gt$

$\therefore V_y = gt$

$\therefore$ velocity at any instant;

$$\therefore \boxed{V = \sqrt{V_x^2 + V_y^2} = \sqrt{u^2 + g^2t^2}}$$

If θ be the angle made by velocity with horizontal velocity, then,

$$\tan\theta = \frac{V_y}{V_x}$$

$$\text{or, } \tan\theta = \left(\frac{gt}{u}\right)$$

$$\boxed{\therefore \theta = \tan^{-1}\left(\frac{gt}{u}\right)}$$

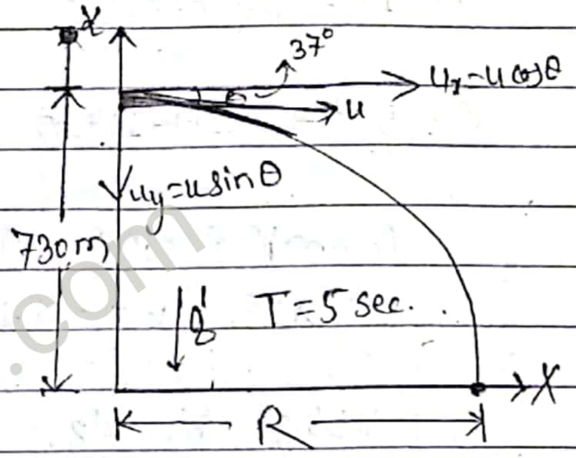
Numericals ⟶

LQ.3[A] An aeroplane diving at an angle of 37° with the horizontal drops a mail bag at an height of 730 m. The projectile hits the ground 5 sec after being realeased. What is the speed of the air craft?

☆ We have;

$h = u_y T + \frac{1}{2} g T^2$

or, $730 = u \cdot \sin 37° \cdot 5 + \frac{1}{2} \times 10 \times 5^2$



or, $730 - 125 = 5u \sin 37°$

or, $121 = u \sin 37°$

or, $u = \frac{121}{\sin 37°}$

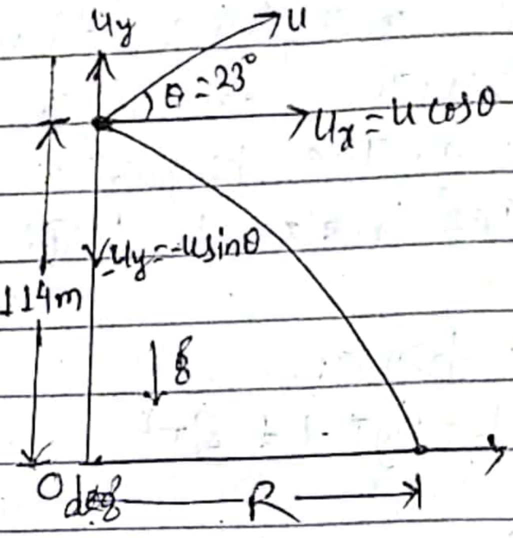
$\therefore u = 201.05 \ ms^{-1}$

Hence, speed of the air craft is $201.05 \ ms^{-1}$. #

LQ.3[B] An air plane is flying with a velocity of $90 \ ms^{-1}$ at an angle of 23° above the horizontal. When the plane is 114 m directly above or dog that is standing on level ground. A sout-case drops out of loadge compartment. How far from the dog will the sout-case land. You can ignore air resistan-ce.

☆ height (h) = 114 m., intial velocity (u) = $90 \ ms^{-1}$, angle of projection (θ) = 23°, $u_x = u\cos\theta$, $u_y = u\sin\theta$, $-u_y = -u\sin\theta$

We have,



$h = -u_y T + \frac{1}{2} \times g \times T^2$

or, $114 = -90 \cdot \sin 23° T + \frac{1}{2} \cdot 10 \times T^2$

or, $5T^2 - 35.16T - 114 = 0$ --- (i)

Comparing eq^n (i) with $ax^2 + bx + c = 0$

$\therefore a = 5, \quad b = -35.16, \ \& \ c = -114$

So,

Using formula,

$$T = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

or, $T = \dfrac{35.16 \pm \sqrt{(-35.16)^2 - 4(5)(-114)}}{2 \times 5}$

or, $T = \dfrac{35.16 \pm \sqrt{3516.63}}{10}$

or, $T = \dfrac{35.16 \pm 59.3}{10}$

Taking +ve,

$T = \dfrac{35.16 + 59.3}{10} \approx 9.446$ sec

$\boxed{\therefore T = 9.4 \text{ sec}}$

Putting $T = 9.4$ sec,

$R = u_x T$

$= u \cos\theta \, T$

$= 90 \cdot \cos 23° \cdot 9.4$

$\therefore \boxed{R = 782.55 \text{ m}}$

Hence, 782.55 m far from the dog will the shoot-case land in 9.4 sec time taken.

LQ.3[c] A body is projected horizontally from the top of a tower 100 m high with a velocity of $9.8\ ms^{-1}$. Find the velocity with which it's hits the ground?

☆ Given;

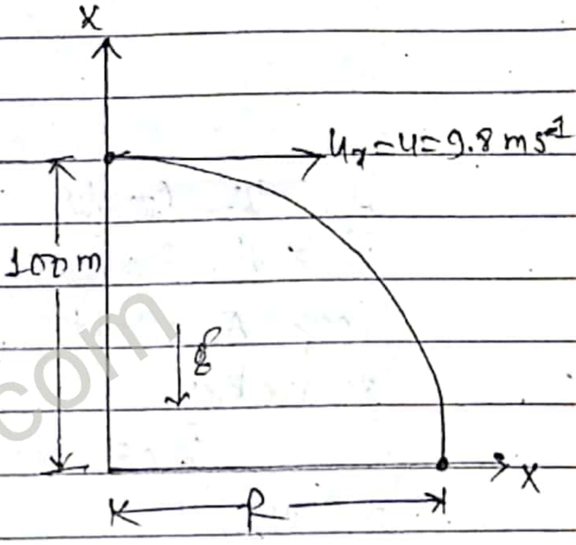
$h = 100m$,

Intial velocity $(u_x) = u = 9.8\ ms^{-1}$

Now,

$T = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 100}{10}} = \sqrt{20}$

$\therefore \boxed{T = 4.47\ sec}$



We know, in the case of velocity at any instant;

$V = \sqrt{V_x^2 + V_y^2}$

$= \sqrt{u^2 + (gt)^2}$

$= \sqrt{(9.8)^2 + (10 \times 4.47)^2}$

$= \sqrt{2094.13}$

$\therefore \boxed{\text{velocity} = 45.76\ ms^{-1}}$

moreover;

$\tan\theta = \frac{V_y}{V_x}$

or, $\tan\theta = \frac{gt}{u}$

or, $\theta = \tan^{-1}\left[\frac{10 \times 4.47}{9.8}\right]$

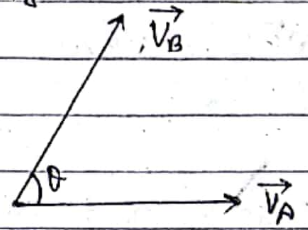
or, $\theta = \tan^{-1}(4.56)$

$\therefore \boxed{\theta = 77.63^\circ}$

Hence, velocity at this instant is 45.76 and direction is 77.63°.

Relative velocity:-

↳ The velocity of one body related to another body is called relative velocity.



Let us consider two bodies A & B moving with velocities $\vec{V_A}$ & $\vec{V_B}$ inclined at an angle θ. Now the velocity of body A related to body B is denoted by $\vec{V_{AB}}$ and given by;

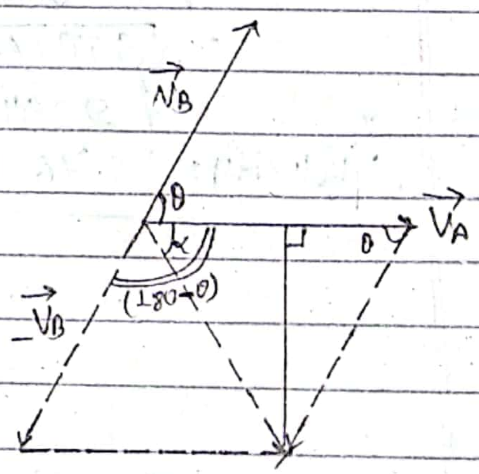
$$\vec{V_{AB}} = \vec{V_A} - \vec{V_B}$$

Similarly,
Velocity of body B related to body A is denoted by $\vec{V_{BA}}$ & defined as,

$$\vec{V_{BA}} = \vec{V_B} - \vec{V_A}$$

★ To find $\vec{V_{AB}}$:

Let us consider reverse $\vec{V_B}$ and complete the parallelogram then diagonal represents $\vec{V_{AB}}$.



Now,
using // gm law of vector addition;

$$\vec{V_{AB}} = \sqrt{V_A^2 + V_B^2 + 2V_AV_B \cdot \cos(180^\circ - \theta)}$$

$$= \sqrt{V_A^2 + V_B^2 - 2V_AV_B\cos\theta}$$

★ $\boxed{\text{Case I}}$: When two bodies are in Same direction, i.e, ($\theta = 0°$)

★

$$\vec{V_{AB}} = \sqrt{V_A^2 + V_B^2 - 2V_AV_B \cos 0°}$$
$$= \sqrt{V_A^2 + V_B^2 - 2V_AV_B}$$
$$= \sqrt{(V_A - V_B)^2}$$
$$\therefore \vec{V_{AB}} = V_A - V_B$$

Hence, when two bodies are in Same direction then resultant is $V_A - V_B$ or decrease.

★ $\boxed{\text{Case II}}$: When two bodies are in opposite direction; i.e, ($\theta = 180°$)

★

$$\vec{V_{AB}} = \sqrt{V_A^2 + V_B^2 - 2V_AV_B \cos 180°}$$
$$= \sqrt{V_A^2 + V_B^2 + 2V_AV_B}$$
$$= \sqrt{(V_A + V_B)^2}$$
$$\therefore \vec{V_{AB}} = V_A + V_B$$

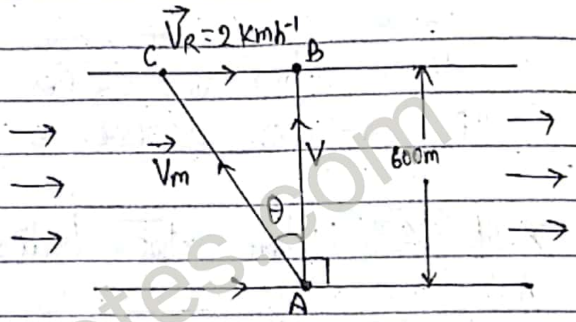
Hence, when two bodies are in opposite direction then resultant or relative velocity is $V_A + V_B$ or increase.

Numerical →

LQ. [4(A)] A man wishes to swim across the river 600 m. wide. If he can swim at the rate of 4 kmh^{-1} in still water and the river flows at 2 kmh^{-1}. Then in what direction must swim to reach a point exactly opposite to the starting point and when will he reach it?

☆ Solution:

Let, Width of river (d) = 600 m
Velocity of man ($\vec{V_m}$) = 4 kmh^{-1}
Velocity of river ($\vec{V_R}$) = 2 kmh^{-1}
direction of man (θ) = ?
Time to cross river (t) = ?



From figure, (In ΔABC)

$$\sin\theta = \frac{BC}{AC} = \frac{V_R}{V_m} = \frac{2}{4} = \frac{1}{2}$$

$$\therefore \theta = \sin^{-1}(1/2) = 30°$$

So, A man can swim by making angle ($90° + 30° = 120°$) with flow of river.

Again,

$V = V_m \cos\theta = 4 \cos 30° = 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}\ kmh^{-1} = \frac{2 \times 1000}{60 \times 60} \times \sqrt{3} = 0.96\ ms^{-1}$

$d = 600$ m

Now, $V = d/t$

or, $t = d/V = \frac{600}{0.96} = 625$ sec

Hence, after 625 second he will reach it.

LQ. [4B] A swimmer's speed along the river (down stream) is 20 kmh^{-1} and can swim up-stream at 8 kmh^{-1}. Calculate the velocity of the stream and the swimmer's possible speed in still water?

Solution:

Let, velocity of River = $\vec{V_R}$

velocity of Swimmer = $\vec{V_S}$

Now,

Along the river (down stream),

$V_R + V_S = 20\ kmh^{-1}$ —— (i)

Along the river (up-stream);

$V_S - V_R = 8\ kmh^{-1}$ —— (ii)

Again; adding eq^n (i) and (ii);

$$V_S + V_R = 20$$
$$V_S - V_R = 8$$
$$2V_S = 28$$
$$\Rightarrow V_S = 14\ kmh^{-1}$$

putting $V_S = 14$, in eq^n (i)

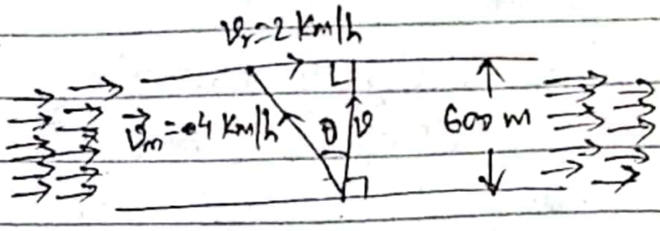
$$V_R + 14 = 20$$
$$\Rightarrow V_R = 6\ kmh^{-1}$$

Thus, velocity of stream (river) is 6 kmh^{-1} and possible speed of the swimmer is 14 kmh^{-1}. #

Page :
Date : / /

Q.4 A man wishes to swim across a river 600m wide. If he can swim at the rate of 4 km/h in still water & the river flows at 2 km/h. Then in what direction must he ~~should to~~ swim to reach a point exactly opposite to the starting point & when will he reach it?

★ Soln:-



$\sin\theta = \frac{v_r}{v_m} = \frac{2}{4} = \frac{1}{2} = \sin 30°$

$\Rightarrow \boxed{\theta = 30°}$

Thus,
He must be run $(30° + 90°) = 120°$ with the water. #

&

$v = (v_m)^2 - (v_r)^2 = (4)^2 - (2)^2 = \sqrt{16-4} = \sqrt{12}$ Km/h

$= \frac{\sqrt{12} \times 1000 m}{60 sec}$

$v = \frac{d}{t}$

$\Rightarrow v = 57.73$ m/min.

& $d = 600$ m.

$t = ?$ Now, $t = \frac{d}{v} = \frac{600}{57.73} = 10.39$ min. #