Unit 2

Vectors

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* Physical Quantities

Vector quantities | Scalar quantities

* **Vector quantities :-**

→ The physical quantities which have both magnitude and direction are called vector quantities.

→ Vector quantities can be added or subtracted according to the rule of vector additions.

→ for e.g - Displacement, velocity, force, Torque, etc.

→ A vector quantities is represented by any alphabet with an arrow head over it ($\vec{A}$) or Bold letter (**A**).

→ A vector is represented by a straight line with arrow at one end. The direction of arrow represents the direction of the vector and Length of line represents the magnitude of vector.

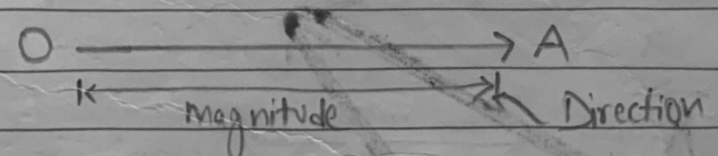


fig : Vector

* Scalar quantities :-

→ The physical quantities which have magnitude only but not direction are called scalar quantities.

→ Scalar quantities can be added or substracted according to the rule of algebra.

→ for e.g. Speed, distance, time, mass, temperature, etc.

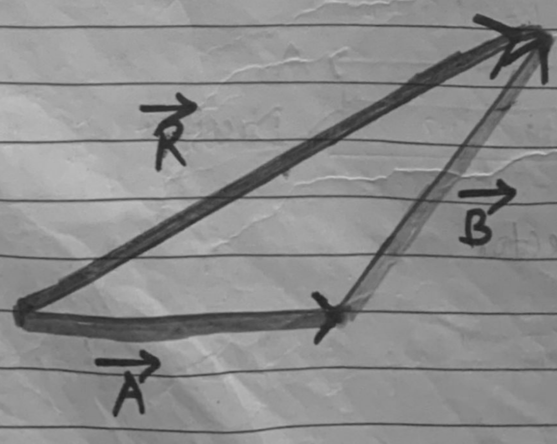
→ Scalar may be positive or negative.

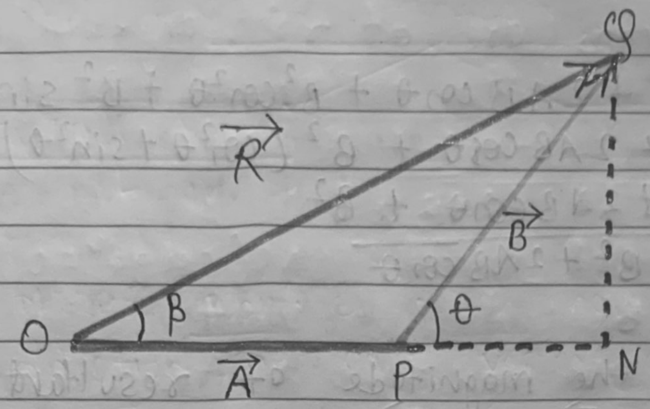
* Addition of vector (Composition of vector) :-

→ The process of obtaining the resultant or sum of the vectors is called addition or composition of vector.

a) Triangle law of vector addition :-

→ If two vectors $\vec{A}$ and $\vec{B}$ ~~are~~ taken in the same order are represented by two sides of a triangle, then the third side of the triangle, taken in opposite order, gives the resultant of vectors $\vec{A}$ and $\vec{B}$.





In right angled triangle PNQ,

$$\cos\theta = \frac{PN}{PQ} = \frac{PN}{B}$$

$$\therefore PN = B\cos\theta$$

$$\sin\theta = \frac{QN}{PQ} = \frac{QN}{B}$$

$$\therefore QN = B\sin\theta$$

In right angled triangle QNO,

$$h^2 = p^2 + b^2$$

or; $OQ^2 = QN^2 + ON^2$

~~or, $OQ^2 = (B\sin\theta)^2 +$~~

or, $OQ^2 = QN^2 + (OP + PN)^2$

or $OQ^2 = OP^2 + 2OP.PN + PN^2 + QN^2$

From, the above figure, in right angled triangle QNO,
$OQ = |\vec{R}| = R$, $OP = |\vec{A}| = A$ and $PQ = |\vec{B}| = B$

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Now,

$$R^2 = A^2 + 2AB\cos\theta + B^2\cos^2\theta + B^2\sin^2\theta$$

or $R^2 = A^2 + 2AB\cos\theta + B^2(\cos^2\theta + \sin^2\theta)$

or $R^2 = A^2 + 2AB\cos\theta + B^2$

$\therefore R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

This gives the magnitude of resultant of vectors $\vec{A}$ and $\vec{B}$

Direction

Let the resultant $\vec{R}$ makes angle β with vector $\vec{A}$

In right angled triangle QNO,

$$\tan\beta = \frac{QN}{ON} = \frac{QN}{OP+PN}$$

Since, $OP = A$, $PN = B\cos\theta$ ~~and B~~ and $QN = B\sin\theta$

Then,

we get,

$$\tan\beta = \frac{B\sin\theta}{A+B\cos\theta}$$

$$\therefore \beta = \tan^{-1}\left(\frac{B\sin\theta}{A+B\cos\theta}\right)$$

This gives the direction of resultant $\vec{R}$ of vectors $\vec{A}$ and $\vec{B}$ with $\vec{A}$.

Special cases:

i) When the two vectors are in the same direction, then $\theta = 0^\circ$ such that,

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$
$$R = \sqrt{A^2 + B^2 + 2AB} \qquad (\because \cos 0^\circ = 1)$$
$$\boxed{R = A + B}$$

and

$$\beta = \tan^{-1}\left(\frac{B\sin 0}{A + B\cos\theta}\right)$$

$$\beta = \tan^{-1}\left(\frac{B \times 0}{A + B\cos\theta}\right) \qquad (\because \sin 0^\circ = 0)$$

$$\beta = \tan^{-1} 0$$

$$\boxed{\beta = 0}$$

ii) When the two vectors are in the opposite direction, then $\theta = 180^\circ$, such that,

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$
$$R = \sqrt{A^2 + B^2 - 2AB} \qquad (\cos 180^\circ = -1)$$

and

$$\beta = \tan^{-1}\left(\frac{B\sin 0}{A + B\cos\theta}\right)$$

$$\beta = \tan^{-1} 0 \qquad (\because \sin 180^\circ = 0)$$
$$\beta = 0$$

iii) When two vectors are at right angles, then $\theta = 90°$ so that,

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$
$$R = \sqrt{A^2 + B^2 + 0} \quad (\cos 90° = 0)$$
$$R = A + B$$

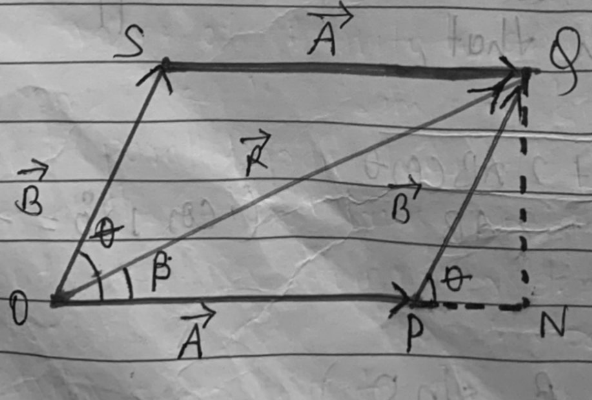
and

$$\beta = \tan^{-1}\left(\frac{B\sin 90°}{A + B\cos 90°}\right) \quad \left(\because \begin{matrix}\sin 90° = 1 \\ \cos 90° = 0\end{matrix}\right)$$

$$= \tan^{-1}\left(\frac{B}{A}\right)$$

b) Parallelogram law of vector addition:-

→ If two vectors $\vec{A}$ and $\vec{B}$ are drawn from the same point and are adjacent to each other, then the diagonal of the parallelogram ~~passing through~~ formed by these vectors represents the resultant of vector $\vec{A}$ and $\vec{B}$.



Magnitude

In right angled triangle QNP

$$\cos\theta = \frac{PN}{PQ} = \frac{PN}{B}$$

$$\therefore PN = B\cos\theta \quad \text{------} \quad \text{eqn (i)}$$

$$\sin\theta = \frac{QN}{PQ} = \frac{QN}{B}$$

$$\therefore QN = B\sin\theta \quad \text{------} \quad \text{eqn (ii)}$$

In right angled triangle ΔQNO;

$$h^2 = p^2 + b^2$$

or, $OQ^2 = QN^2 + ON^2$

or, $OQ^2 = QN^2 + (OP+PN)^2$

or, $OQ^2 = QN^2 + OP^2 + 2\,OP.PN + PN^2$ ------ (iii)

from the right angled triangle ΔQNO, $OQ = |\vec{R}| = R$, $OP = |\vec{A}| = A$ and $PQ = |\vec{B}| = B$

By using eqn (i), (ii) and (iii) can be written as,

$$R^2 = B^2\sin^2\theta + A^2 + 2AB\cos\theta + B^2\cos^2\theta$$

$$R^2 = B^2\sin^2\theta + B^2\cos^2\theta + A^2 + 2AB\cos\theta$$

$$R^2 = B^2(\sin^2\theta + \cos^2\theta) + A^2 + 2AB\cos\theta$$

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

This gives the magnitude of resultant of vectors $\vec{A}$ and $\vec{B}$

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Direction :

we suppose the resultant $\vec{R}$ makes angle β with vector $\vec{A}$.

In right angled triangle ΔQNO,

$$\tan\beta = \frac{QN}{ON}$$

$$\tan\beta = \frac{B\sin\theta}{OP+PN}$$

$$\tan\beta = \frac{B\sin\theta}{A+B\cos\theta}$$

$$\beta = \tan^{-1}\left(\frac{B\sin\theta}{A+B\cos\theta}\right)$$

Thus, this gives the direction of resultant R of vectors $\vec{A}$ and $\vec{B}$

* The value of magnitude and direction of (R) is the same is the same in both cases of triangle law and parallelogram law of vectors. Therefore any of the two laws apply to the same problems. But when solving problems, one of them is chosen so that the vector diagram can be drawn easily according to the given conditions.

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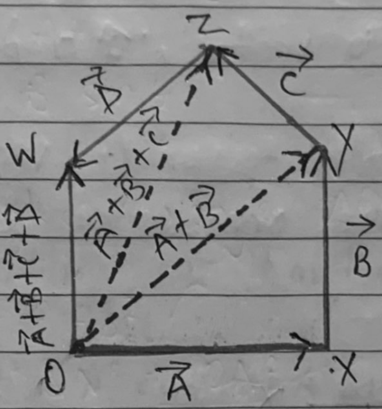
c) Polygon law of vector addition :-

→ If multiple vectors are arranged in sequence, the resultant vector is represented by a straight line drawn from the initial point to the final point in the opposite order, then it is known as polygon law of vector addition.



* Subtraction of a vector :-

→ The subtraction of a vector from another vector is the same as the addition of a negative vector with another vector.

→ If vector $\vec{B}$ is to be subtracted from another vector $\vec{A}$, then it is given by,

$$\vec{A}+(-\vec{B})=\vec{A}-\vec{B}$$

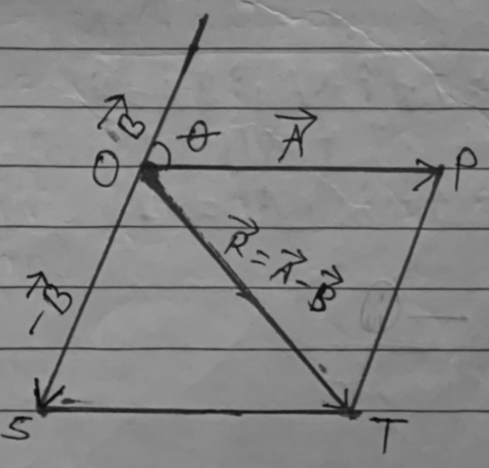


fig :- Subtraction of vector

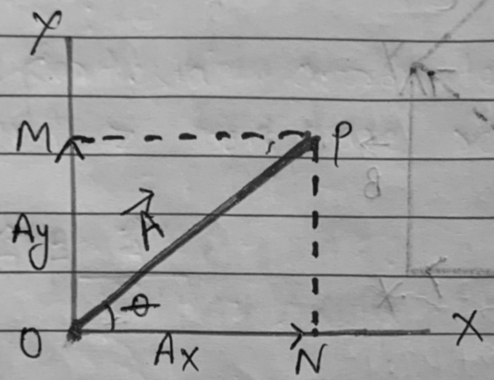
→ Resultant

$$R = \sqrt{A^2 + 2AB \cos(180-\theta) + B^2}$$

$$R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

✱ Resolution of a vector :-

→ The process of spliting of vectors into it's component is called resolution of a vector.



→ Let θ be the angle made by $\vec{A}$ with x-axis.

Now,

In right angled △ PON,

$$\sin\theta = \frac{PN}{PO}$$

or, $\sin\theta = \frac{PN}{A}$

or, $PN = A\sin\theta$

or, $OM = A\sin\theta$

or, $A_y = A\sin\theta$ — (i)

Again,
In right angled Δ PMO,

$$\cos\theta = \frac{PM}{PO}$$

or, $\cos\theta = \frac{NO}{A}$

or, $A_x = A \cos\theta$ ---- (ii)

squaring & adding eqn (i) and (ii)

$$A_x^2 + A_y^2 = A^2\cos^2\theta + A^2\sin^2\theta$$

or, $A_x^2 + A_y^2 = A^2(\cos^2\theta + \sin^2\theta)$

or, $A^2 = A_x^2 + A_y^2$

or, $A = \sqrt{A_x^2 + A_y^2}$

Again, Dividing eqn (i) by (ii)

$$\frac{A\sin\theta}{A\cos\theta} = \frac{A_y}{A_x}$$

$$\tan\theta = \frac{A_y}{A_x}$$

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

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* **Unit vector:-**

→ A vector whose magnitude are 1 are called unit vectors,

→ Unit vector's are represented by (Cap/Hat) symbol on top of any letter.

for eg. If $\vec{a}$ is a vector, $\hat{a}$ represent a unit vector.

→ In cartesian coordinates, unit vector along x, y and z axis are represented by $\hat{i}, \hat{j}$ and $\hat{k}$.

→ $$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

$$|\vec{A}| = \sqrt{A^2x + A^2y + A^2z}$$ (comparing with $\vec{A} = Ax\,\hat{i} + Ay\,\hat{j} + Az\,\hat{k}$)

Numerical.

* Find the unit vector of $3\hat{i} + 4\hat{j} - \hat{k}$.

→ sol> Let,

$$\vec{A} = 3\hat{i} + 4\hat{j} - \hat{k}$$

comparing with,

$$\vec{A} = Ax\,\hat{i} + Ay\,\hat{j} + Az\,\hat{k}$$

$Ax = 3$, $Ay = 4$, $Az = -1$

Also,

$$|\vec{A}| = \sqrt{A^2x + A^2y + A^2z}$$

$|\vec{A}| = \sqrt{3^2 + 4^2 + (-1)^2}$

$|\vec{A}| = \sqrt{9 + 16 + 1}$

$|\vec{A}| = \sqrt{26}$ units

Now,

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

$$\hat{A} = \frac{3\hat{i} + 4\hat{j} - \hat{k}}{\sqrt{26}}$$

$$\hat{A} = \frac{3\hat{i}}{\sqrt{26}} + \frac{4\hat{j}}{\sqrt{26}} - \frac{1\hat{k}}{\sqrt{26}}$$ #

* Multiplication of vector (Product of vector) :-

i) Scalar product :-

→ When two vectors are multiplied and the result obtained is a scalar quantity, then the multiplication is known as scalar product.

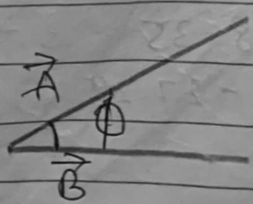
→ It is also known as dot product.

→ suppose, we have two vectors, $\vec{A}$ and $\vec{B}$ and the angle between them is ϕ, then the scalar product of $\vec{A}$ and $\vec{B}$ is defined as,

$$\vec{A}\cdot\vec{B} = |\vec{A}| \cdot |\vec{B}| \cos\phi$$

or,

$$A \cdot B \cos\phi$$



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→ Scalar product of $\vec{A}$ and $\vec{B}$ is represented as $\vec{A} \cdot \vec{B}$. So, it is also known as dot product.

→ for e.g. work, electric flux, etc.

QSNS

* If $\vec{A} = (2, -5)$ and $\vec{B} = (4, 7)$ then what is $\vec{A} \cdot \vec{B}$?

→ Soln Given,

$\vec{A} = (2, -5)$
$\vec{B} = (4, 7)$

we know that,

$\vec{A} \cdot \vec{B} = A_x \cdot B_x + A_y \cdot B_y$

So,

$\vec{A} \cdot \vec{B} = 2 \times 4 + (-5 \times 7)$
$= 8 - 35$
$= -27$ #

* If $\vec{A} = (2\hat{i} - 5\hat{j})$ and $\vec{B} = (4\hat{i} + 7\hat{j})$ the find $\vec{A} \cdot \vec{B}$.

→ Soln Given,

$\vec{A} = (2\hat{i} - 5\hat{j})$
$\vec{B} = (4\hat{i} + 7\hat{j})$

now,

$\vec{A} \cdot \vec{B} = 2\hat{i}(4\hat{i} + 7\hat{j}) - 5\hat{j}(4\hat{i} + 7\hat{j})$
$= 8\,\hat{i}\cdot\hat{i} + 14\,\hat{i}\cdot\hat{j} - 20\,\hat{j}\cdot\hat{i} - 35\,\hat{j}\cdot\hat{j}$
$= 8 \times 1 + 14 \times 0 - 20 \times 0 - 35 \times 1$
$= 8 + 0 - 0 - 35$
$= 8 - 35$
$= -27$ #

ii) **Vector product :-**

→ When two vectors are multiplied and the result obtained is a vector quantity, then the multiplication is known as a vector product.

→ Suppose, we have two vectors $\vec{A}$ and $\vec{B}$ and the angle between them is ϕ, then the vector product of $\vec{A}$ and $\vec{B}$ is represented by $|\vec{A} \times \vec{B}|$

$$|\vec{A} \times \vec{B}| = |\vec{A}| \cdot |\vec{B}| \sin\phi \, \hat{n}$$

Numerical

* If the magnitude of $\vec{A}$ is 5 and the magnitude of $\vec{B}$ is 6 and the angle between $\vec{A}$ and $\vec{B}$ is $30°$, what is the magnitude of $\vec{A} \times \vec{B}$?

→ Sol^n

Given,

$\vec{A} = 5$

$\vec{B} = 6$

$\phi = 30°$

$\vec{A} \times \vec{B} = ?$

now,

$|\vec{A} \times \vec{B}| = \vec{A} \cdot \vec{B} \cdot \sin\phi \, \hat{n}$

$= 5 \cdot 6 \cdot \sin 30°$

$= 5 \cdot \cancel{6}^{3} \cdot \frac{1}{\cancel{2}}$

$= 15$ square units

→ It is also known as cross product.

→ for eg- ~~Circular motion~~ Torque, Angular momentum, etc

Special Cases for scalar product

i) While two vectors are perpendicular
Then, $\theta = 90^\circ$, $\cos 90^\circ = 0$

$$\vec{A}\cdot\vec{B} = |\vec{A}|.|\vec{B}| \cos 90^\circ$$
$$\vec{A}\cdot\vec{B} = |\vec{A}|.|\vec{B}| \times 0$$
$$\vec{A}\cdot\vec{B} = 0$$

ii) While two vectors are parallel
Then, $\theta = 0^\circ$, $\cos 0^\circ = 1$

$$\vec{A}\cdot\vec{B} = |\vec{A}|.|\vec{B}| \cos 0^\circ$$
$$\vec{A}\cdot\vec{B} = |\vec{A}|.|\vec{B}| \text{ or } AB$$

iii) While two vectors are anti parallel
Then, $\theta = 180^\circ$, $\cos 180^\circ = -1$

$$\vec{A}\cdot\vec{B} = |\vec{A}|.|\vec{B}| \cos 180^\circ$$
$$\vec{A}\cdot\vec{B} = -|\vec{A}|.|\vec{B}| \text{ or } -AB$$

Properties of scalar product

→ The scalar product of two vector obey commutative law
i.e. $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

→ Scalar product of two vector obey distributive law
i.e. $\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$

→ The square of vector is equal to square of its magnitude.

i.e. $(\vec{A})^2 = \vec{A} \cdot \vec{A} = |\vec{A}||\vec{A}| \cos 0 = A^2$

Vector Product Numerical.

* $\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{B} = 2\hat{i} + 4\hat{j} - 5\hat{k}$

Find $|\vec{A} \times \vec{B}|$

→ sol^n

$\vec{A} \times \vec{B} = (\hat{i} + 2\hat{j} + 3\hat{k})(2\hat{i} + 4\hat{j} - 5\hat{k})$

$= 2\hat{i} \times \hat{i} + 4\,\hat{i} \times \hat{j} - 5\,\hat{i} \times \hat{k} + 4\,\hat{j} \times \hat{i} + 8\,\hat{j} \times \hat{j} - 10\,\hat{j} \times \hat{k} + 6\,\hat{k} \times \hat{i} + 12\,\hat{k} \times \hat{j} - 15\,\hat{k} \times \hat{k}$

$= 2 \times 0 + 4k + 5\hat{j} - 4\hat{k} + 8 \times 0 - 10\hat{i} + 6\hat{j} - 12\hat{i} - 15 \times 0$

$= 0 + \cancel{4\hat{k}} - \cancel{4\hat{k}} + 5\hat{j} + 6\hat{j} - 10\hat{i} - 12\hat{i}$

$= 11\hat{j} - 22\hat{i}$

Numericals

* A force of 6N act on a body due to east and other force 8N act due to north. What is resultant force.

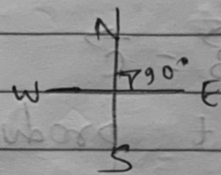
Soln

we have,

Force acting on east $(F_1) = 6N$

force acting on north $(F_2) = 8N$

Angle $(\theta) = 90^\circ$



Now,

$$\text{Resultant } (R) = \sqrt{(F_1)^2 + (F_2)^2 + 2(F_1)(F_2)\cos\theta}$$

$$= \sqrt{6^2 + 8^2 + 2 \times 6 \times 8 + \cos 90^\circ}$$

$$= \sqrt{36 + 64 + 96 + 0}$$

$$= 10$$

$$\text{Direction } (\beta) = \tan^{-1}\left(\frac{F_2}{F_1}\right)$$

$$= \tan^{-1}\left(\frac{8}{6}\right)$$

$$= \tan^{-1}\left(\frac{4}{3}\right)$$

$$= 53.13^\circ \text{ \#}$$

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* Two vectors A and B are such that $\vec{A} - \vec{B} = \vec{C}$ and $A - B = C$, find the angle between them.

⇒ Soln Here,

$\vec{A} - \vec{B} = \vec{C}$

$A - B = C$

Now,

$\vec{A} - \vec{B} = \vec{C}$

self dot product on both sides

$(\vec{A} - \vec{B}) \cdot (\vec{A} - \vec{B}) = (\vec{C})(\vec{C})$

or $\vec{A} \cdot \vec{A} - \vec{A} \cdot \vec{B} - \vec{B} \cdot \vec{A} + \vec{B} \cdot \vec{B} = C^2$

or $\vec{A} \cdot \vec{A} - \vec{A} \cdot \vec{B} - \vec{A} \cdot \vec{B} + \vec{B} \cdot \vec{B} = C^2$

or $A^2 - 2AB\cos\theta + B^2 = C^2$ — (i)

$A - B = C$

Squaring both sides,

$(A-B)^2 = C^2$

or $A^2 - 2AB + B^2 = C^2$ — (ii)

Comparing eqn (i) and (ii), we get

$2AB = 2AB\cos\theta$

or $\cos\theta = \frac{2AB}{2AB}$

or $\cos\theta = 1$

or $\theta = 0°$ #

* If the magnitude of two vectors are 3 and 4 and magnitude of their scalar product is 6. find the angle between the vectors.

⇒ soln

Let,

$|\vec{A}| = 3$

$|\vec{B}| = 4$

$\vec{A} \cdot \vec{B} = 6$

now,

$\vec{A} \cdot \vec{B} = |\vec{A}| \cdot |\vec{B}| \cdot \cos\theta$

or $6 = 3 \times 4 \cos\theta$

or $\frac{6}{12} = \cos\theta$

or $\cos\theta = \frac{1}{2}$

$\therefore \theta = 60^\circ$

* If the scalar product of two vector is equal to the magnitude of their vector product.

⇒ soln

If ~~scalar~~ $\vec{A}$ and $\vec{B}$ are two vectors,

Scalar product of two vectors.

$\vec{A} \cdot \vec{B} = AB \cos\theta$ — (i)

Vector product of two vectors

$\vec{A} \times \vec{B} = AB \sin\theta \ \hat{n}$ —.

According to qsn,

$\vec{A} \cdot \vec{B} = |\vec{A} \times \vec{B}|$

or $\cancel{A}B \cos\theta = \cancel{A}B \sin\theta$

or $1 = \frac{\sin\theta}{\cos\theta}$

or $1 = \tan\theta$

or $\theta = 45^\circ$

* If $\vec{A} \cdot \vec{B} = 0$, what is the angle between $\vec{A}$ and $\vec{B}$.

→ soln

$\vec{A} \cdot \vec{B} = 0$

$\theta = ?$

now,

$\vec{A} \cdot \vec{B} = AB \cos\theta$

or $0 = AB \cos\theta$

or, $\cos\theta = \frac{0}{AB}$

or $\cos\theta = 0$

or $\theta = 90^\circ$ #

$\theta = 0$

* $\vec{C}$ is the sum of $\vec{A}$ and $\vec{B}$ that is $\vec{C} = \vec{A} + \vec{B}$, for $C = A + B$ to be true. What is the angle between $\vec{A}$ and $\vec{B}$.

⇒ Soln Given

$$\vec{A} + \vec{B} = \vec{C}$$

$$A + B = C$$

now,

$$\vec{A} + \vec{B} = \vec{C}$$

self dot product on both sides.

$$(\vec{A} + \vec{B}) \cdot (\vec{A} + \vec{B}) = \vec{C} \cdot \vec{C}$$

or $\vec{A} \cdot \vec{A} + \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{B} + \vec{B} \cdot \vec{B} = C^2$

or $A^2 + 2AB\cos\theta + B^2 = C^2$ — (i)

Again,

$$A + B = C$$

Squaring on both sides

$$(A+B)^2 = C^2$$

or $A^2 + 2AB + B^2 = C^2$ — (ii)

Comparing eqn (i) & (ii)

$$2AB\cos\theta = 2AB$$

$$\cos\theta = 1$$

$$\theta = 0^\circ$$ #

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* If $\vec{A}$ and $\vec{B}$ are non 0 vectors. It is possible for ~~vector~~ $\vec{A} \times \vec{B}$ and $\vec{A} \cdot \vec{B}$ both 0? Explain.

⇒ sol when the angle between $\vec{A}$ and $\vec{B}$ is 0.

Yes,

$\vec{A} \times \vec{B} = AB \sin\theta \, \hat{n}$

$\vec{A} \times \vec{B} = AB \sin 0 \, \hat{n}$

$\vec{A} \times \vec{B} = 0$

When the angle between $\vec{A} \cdot \vec{B}$ is 90°

Then,

$\vec{A} \cdot \vec{B} = AB \cos\theta$

$\vec{A} \cdot \vec{B} = AB \cos 90^\circ$

$\vec{A} \cdot \vec{B} = 0$

It is possible when value of θ is 0° and 90° in $\vec{A} \times \vec{B}$ and $\vec{A} \cdot \vec{B}$ respectively.

* Two vector have equal magnitudes & their resultant also have same magnitude, What is the angle between two vectors.

⇒ soln

Let $\vec{P}$ and $\vec{Q}$ be the two vectors and $\vec{R}$ be the resultant.

According to qsn.

θ be the angle between $\vec{P}$ and $\vec{Q}$

Triangle law of vector addition,

$$R = \sqrt{P^2+Q^2+2PQ\cos\theta}$$

$$R = \sqrt{R^2+R^2+2R^2\cos\theta}$$

$$R = \sqrt{2R^2+2R^2\cos\theta}$$

$$R = R\sqrt{2+2\cos\theta}$$

$$1 = \sqrt{2+2\cos\theta}$$

Squaring both sides

$$1^2 = (\sqrt{2+2\cos\theta})^2$$

or $1 = 2+2\cos\theta$

or $\dfrac{1-2}{2} = \cos\theta$

or, $\cos\theta = \dfrac{-1}{2}$

$\therefore$ $\theta = 120°$ #

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* Given two $\vec{A} = 4.00\hat{i} + 3.00\hat{j}$ and $\vec{B} = 5.00\hat{i} - 2.00\hat{j}$

⇒ sol^n
We have,

$\vec{A} = 4.00\hat{i} + 3.00\hat{j}$

$\vec{B} = 5.00\hat{i} - 2.00\hat{j}$

now,

$|\vec{A}| = \sqrt{4^2+3^2} = \sqrt{16+9} = \sqrt{25} = 5$

$|\vec{B}| = \sqrt{5^2-2^2} = \sqrt{25-4} = \sqrt{21}$

* Find the scalar product of two vectors $\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{B} = 3\hat{i} + 4\hat{j} - 5\hat{k}$ and also find the angle between them

⇒ sol^n
We have,

$\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

$\vec{B} = 3\hat{i} + 4\hat{j} - 5\hat{k}$

now,

$\vec{A}\cdot\vec{B} = (3\hat{i} + 4\hat{j} + 5\hat{k})(3\hat{i} + 4\hat{j} - 5\hat{k})$

$= 3\hat{i}(3\hat{i} + 4\hat{j} - 5\hat{k}) + 4\hat{j}(3\hat{i} + 4\hat{j} - 5\hat{k}) + 5\hat{k}(3\hat{i} + 4\hat{j} - 5\hat{k})$

$= 9\hat{i}.\hat{i} + 12\hat{i}.\hat{j} - 15\hat{i}.\hat{k} + 12\hat{j}.\hat{i} + 16\hat{j}.\hat{j} - 20\hat{j}.\hat{k} + 15\hat{k}.\hat{i} + 20\hat{j}.\hat{k} - 25\hat{k}.\hat{k}$

$= 9\times1 + 12\times0 - 15\times0 + 12\times0 + 16\times1 - 20\times0 + 15\times0 + 20\times0 - 25\times1$

$= 9 + 16 - 25$

$= 0$

$$|\vec{A}| = \sqrt{3^2+4^2+5^2}$$
$$= \sqrt{9+16+25}$$
$$= \sqrt{50}$$
$$= 5\sqrt{2}$$

$$|\vec{B}| = \sqrt{3^2+4^2+(-5)^2}$$
$$= \sqrt{25+25}$$
$$= 5\sqrt{2}$$

so,

$$\vec{A}\cdot\vec{B} = |\vec{A}|\cdot|\vec{B}|\cdot\cos\theta$$

or, $$\frac{0}{(5\sqrt{2})\times(5\sqrt{2})} = \cos\theta$$

or $\theta = \cos^{-1} 0$

$\therefore \theta = 90°$ #

Imp ★ A force expressed in a vector ~~not~~ notation $\vec{F} = 4\hat{i} + 7\hat{j} - 3\hat{k}$ is applied on a body and produce a displacement $(\vec{D})$ $= 3\hat{i} - 2\hat{j} - 5\hat{k}$ in 4s, estimate the power.

→ Soln we have,

Work $= \vec{F}\cdot\vec{D}$

now,

$$\text{Work} = (4\hat{i}+7\hat{j}-3\hat{k})\cdot(3\hat{i}-2\hat{j}-5\hat{k})$$
$$= 12 - 14 + 15$$
$$= 13 \text{ Joule}$$

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$$\text{Power} = \frac{W}{T}$$

$$= \frac{13}{4}$$

$$= 3.25 \text{ watt.}$$