UNIT 1 MECHANICS

Chapter 1

PHYSICAL QUANTITIES

☆ Measurement

The process of Comparing an unknown physical quantity with a known fixed unit quantity is called measurement.

☆ Physical Quantity

Those quantity which can be measured and can be expressed in numerical value are called physical quantity. For example: Mass, Length, time, etc.
A physical quantity is expressed as;
physical quantity = Numerical Value(N) × Unit (U)

The physical quantity can be divided into two parts
(i) Fundamental Quantity
(ii) Derived Quantity

Fundamental Quantity:-

The basic physical quantity which is taken as standard to measure other physical quantities known as fundamental quantity. for example: Length, mass, time, temperature, electric current, Luminous intensity, amount of Chemical Substance.

Derived Quantity :-

A quantity obtained from fundamental quantities is called a derived quantity. For example:- Area, volume, density, speed, electric intensity, etc.

Unit

The standard quantity in terms of which the given physical quantities can be compared is called unit. For example: k.g., second, Newton, etc.

There are two types of unit:-

(i) Fundamental unit

(ii) Derived unit

Fundamental Unit:-

The units of fundamental quantities are called fundamental units. For example:- k.g., m, s, k, etc.

Derived Unit :-

The units of Derived quantities are called derived units. For e.g.:- m/s, m/s^2, etc.

System of Units

i.) F.P.S. System :- In this system length is measured in foot, mass in pound and time in second. This is British system of units. Here, F → foot, P → pound, s → second.

ii.) MKS system:- In this system, Length is measured in metre, mass in K.g., and time in second. This system was developed in France. Here, M→Metre, K→K.g. & S→second.

iii.) CGS system:- In this system Length is measured in centimetre, mass in gram and time in ~~gram~~ second. This system was developed in France. Here, C→centimetre, G→Gram and S→second.

Characteristics of a Standard Unit

i.) It should be well defined and of a suitable size.
ii.) It should be easily available, so that easily reproduce in laboratories.
iii.) It should not change with time and place.
iv.) It should not change with change in physical condition.
v.) It should be universally agreed, so that result of measurement at different places can be compared.

iv.) SI System:-
The internationally standardised unit of measurement is called SI system.

* In SI system of unit, 7 fundamental units and 2 supplementary units were proposed. These units are written below:-

Fundamental Quantities	S.I. Units	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Temperature	kelvin	K
Luminous Intensity	candela	cd
Electric Current	ampere	A
Amount of Chemical Substance	mole	mol
Supplementary quantities		
plane angle	radian	rad
Solid angle	steradian	sr

☆ Dimensions of physical quantity:-

The power raised to the fundamental quantities which are involved in desired physical quantities is called dimension of physical quantity.

☆ Dimensional formula of different physical quantities:-

1) Speed = $\frac{distance}{time} = \frac{[L]}{[T]} = [LT^{-1}]$

∴ the dimensional formula of speed = $[M^0LT^{-1}]$

2) velocity = $\frac{disp.}{time} = \frac{[L]}{[T]} = [LT^{-1}]$

∴ The dimensional formula of Velocity = $[M^0LT^{-1}]$

From above, Both Speed and Velocity has same dimensional formula. But Speed is scalar quantity and velocity is vector quantity. Hence, from dimensional formula we cannot say physical quantity is scalar or vector.

☆ Dimensional equation:-

An eq^n containing physical quantities, each quantity is represented by its dimensional formula, the resulting eq^n is known as dimensional eq^n.

Next difination:
When the dimension of a quantity is found and expressed in the form of an equation, the eq^n is called the dimensional eq^n.

☆ principle of homogenity of dimension:

The pysical relations must obey the principle of homogenity. According to this concept; "every term in a physical relation must have the same dimension."

Suppose in physical relation, $s = ut + \frac{1}{2}at^2$, there are three terms: s, ut & $\frac{1}{2}at^2$. All the terms must have the same dimension, i.e., $[s] = [ut] = [\frac{1}{2}at^2]$, to obey the principle of homogeneity.

☆ Table of Units and Dimensions of physical Quantities:

S.N.	Fundamental physical Quantity	Dimensional Formula	SI Unit of physical Quantity
1.	Mass	[M]	kilogram
2.	Length	[L]	metre
3.	Time	[T]	second
4.	Temperature	[K] or [θ]	kelvin
5.	Electric current	[I] or [A]	ampere
6.	Amount of substance	[N]	mole (mol)
7.	Luminous intensity	[J]	candela (cd)

☆ $A + B = C + D$ / or, $A + B - C - D = 0$

According to Homogenity,

dim. of A = dim. of B = dim. of C = dim. of D.

☆ Derived physical Quantities:

S.No.	Physical Quatity	formula (relation)	Dim. formula	SI Unit
1.	Density	mass/volume	$[M^1 L^{-3} T^0]$	$Kg\,m^{-3}$
2.	Speed or Velocity	distance/time	$[M^0 L^1 T^{-1}]$	m/s
3.	Acceleration	velocity/time	$[M^0 L^1 T^{-2}]$	m/s^2
4.	momentum	mass x velocity	$[M^1 L^1 T^{-1}]$	$Kg\,m\,s^{-1}$
5.	Force	mass x acceleration	$[M^1 L^1 T^{-2}]$	N (Newton)
6.	pressure	Force/area	$[M^1 L^{-1} T^{-2}]$	Nm^{-2} or pa
7.	Work	force x displacement	$[M^1 L^2 T^{-2}]$	J (joule)
8.	Energy	Work ($E=mc^2$)	$[M^1 L^2 T^{-2}]$	J
9.	Gravitational Constant (G)	force x d^2/$(mass)^2$	$[M^{-1} L^3 T^{-2}]$	$Nm^2 kg^{-2}$
10.	Surface tension	force/length	$[M^1 L^0 T^{-2}]$	Nm^{-1}
11.	Moment of inertia	mass x $(distance)^2$	$[M^1 L^2 T^0]$	$Kg\,m^2$
12.	Angular momentum	moment of Intertia x angular velocity	$[M^1 L^2 T^{-1}]$	$kg\,m^2 s^{-1}$
13.	Torque or Couple	force x ⊥r distance	$[M^1 L^2 T^{-2}]$	Nm
14.	Frequency	1/second	$[T^{-1}]$	Hz
15.	Angular velocity (ω)	velocity/radius	$[T^{-1}]$	s^{-1}
16.	Specific Heat	energy/mass x temp^r	$[M^0 L^2 T^{-2} K^{-1}]$	$J\,Kg^{-1}\,°C^{-1}$
17.	Stress	Force/Area	$[M^1 L^{-1} T^{-2}]$	N/m^2
18.	Strain	Δl/l	$[L^0]$ dimensionless	
19.	Refractive Index	V_1/V_2	Dimensionless	
20.	Machenical Advantage	Load/Effort	"	
21.	Electric charge	I x t	$[M^0 L^0 T^1 I^1]$	amp-sec or Coulomb
22.	Electric resistance	V/I	$[M^1 L^2 T^{-3} I^{-2}]$	ohm (Ω) or Volt/amp
23.	Boltzmann's Constant (k)	energy/temperature	$[M^1 L^2 T^{-2} K^{-1}]$	J/K
24.	plank's Constant (h)	$E=hf$	$[M^1 L^2 T^{-1}]$	J.s. (or) eV.s.
25.	power of lens (P)	$P=1/f$	$[L^{-1}]$	dioptre
26.	Angle		Dimensionless	rad

☆ Uses of Dimensional formula:-

✳ To check the correctness of a physical eqⁿ:

Example:

* Check whether the physical eqⁿ $v^2 = u^2 + 2as$ is dimensionally correct or not.

⇒ Solⁿ:

Given formula

$v^2 = u^2 + 2as$

$[L.H.S.] = [v^2] = [LT^{-1}]^2 = [L^2T^{-2}]$ ---(i)

$[R.H.S.] = [u^2 + 2as] = [LT^{-1}]^2 + [LT^{-2}.L] = [L^2T^{-2}] + [L^2T^{-2}]$

Since, the number '2' is dimensionless,

$[R.H.S.] = [L^2T^{-2}] + [L^2T^{-2}]$ ---(ii)

from eqⁿ (i) & (ii), we have,

$[L.H.S.] = [R.H.S.]$

Hence, the given eqⁿ is dimensionally correct.

To derive the relationship between different physical quantities:

* example:

Derive the dimensional relation of time period of pendulum with ~~mass~~, length and acceleration due to to gravity.

⇒ Solⁿ:

$T \propto l^a g^b$

or, $T = K\, l^a g^b$ ---(i)

Where K is dimensionless constant.

Dim. eqⁿ of eqⁿ (i) is,

$[T] = [L]^a [LT^{-2}]^b$

$[M^0L^0T^1] = [M^0L^{a+b}T^{-2b}]$

equating dimension on both sides, we get

(i) $1 = -2b$ or, $b = -1/2$

(ii) $a + b = 0$ or, $a = 1/2$

putting the value of a & b in eq^n (i)

$$T = K\, l^{1/2} g^{-1/2}$$

$$\text{or, } T = K \frac{l^{1/2}}{g^{1/2}}$$

$$\text{or, } T = K\sqrt{\frac{l}{g}}$$ #

To convert a unit from one system into another:

* Example:- Convert 10 dyne into newton.

⇒ Sol^n: Let 10 dyne = N_2 newton.

where, dyne is the unit of force in CGS, and newton in SI.

dim. formula of force is as.

$$[\text{force}] = [MLT^{-2}]$$

∴ a = 1, b = 1 & c = -2 in mass, length & time respectively.

CGS system	SI. system
N_1 = 10 dyne	N_2 = ?
M_1 = 1 g.	M_2 = 1 kg
L_1 = 1 cm	L_2 = 1 m
T_1 = 1 s	T_2 = 1 s

According to conversion formula,

$$N_2 = N_1\left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

Putting above values, we get,

$$N_2 = 10\left[\frac{1g}{1kg}\right]^1 \left[\frac{1cm}{1m}\right]^1 \left[\frac{1s}{1s}\right]^{-2}$$

$$= 10\left[\frac{1g}{1000g}\right] \times \left[\frac{1cm}{100cm}\right] \times \left[\frac{1s}{1s}\right]^{-2}$$

$$\text{or, } 10 \times 10^{-3} \times 10^{-2} \times 1 = 10^{-4}\, N$$

Hence,

10 dyne = 10^{-4} N. #

To determine the dimension of a constant.

* Example1:- Determine the dimension of universal gravitational constant (G)?

⇒ Soln: We have, $F = \frac{G m_1 m_2}{r^2}$

or, $G = \frac{F r^2}{m_1 m_2}$

$= \frac{[MLT^{-2}][L^2]}{[M][M]}$

$= [M^{-1}L^3T^{-2}]$

Hence the dimension of G are -1 in mass, 3 in length and -2 in time.

* Example2: find the dimensional formula of co-efficient of viscosity.

→ Soln: We have, $F = 6\pi\eta r v$

$\eta = \frac{F}{6\pi r v}$

$= \frac{[MLT^{-2}]}{[L][LT^{-1}]}$

$= [ML^{-1}T^{-1}]$

Hence, the dimensional formula of Co-efficient of viscosity is $[ML^{-1}T^{-1}]$ #

To establish relationship between different physical quantities:

* example:- The centripetal force depends upon mass, velocity and radius of circular path.

→ Soln: then,

$$F \propto m^a v^b r^c$$

$$F = K m^a v^b r^c \quad \text{--- (i)}$$

where K is dimensionless constant.

Dimensional eqn of eqn (i), is,

$$[MLT^{-2}] = [M]^a [LT^{-1}]^b [L]^c$$

or, $[MLT^{-2}] = [M^a L^{b+c} T^{-b}]$

equating dimension on both side, we get.

(i) $a = 1$ (ii) $-b = -2$ $\therefore b = 2$ (iii) $b + c = 1$ or, $2 + c = 1$ $\therefore c = -1$ #

$$\Rightarrow F = \frac{Kmv^2}{r} = \frac{mv^2}{r}$$

Q.1(A) If $v = a + bt$ where v is velocity and t is time then find dim. of 'a' & 'b'?

⇒ Soln: $v = a + bt$

or, $v - a - bt = 0$

according to homogenity,

dim. of a = dim. of v

$= [LT^{-1}]$

$\therefore$ dim. of $a = [M^0 L^1 T^{-1}]$ #

dim. of $b \times t$ = dim. of v

$b \times T = L/T$

$b = \frac{L}{T^2} = [LT^{-2}]$

$\therefore$ dim. of b is $[M^0 LT^{-2}]$ #

Q.1 [B] If $x = a + c/m$ where, x = disp., m = mass, and find the dimension of 'a' & 'c'?

→ soln:

We have, $x = a + c/m$

or, $x - a - c/m = 0$

According to homogenity;

Dim. of a = dim. of x
$= L$

$\therefore$ Dim. of $a = [M^0 L^1 T^0]$ #

Dim. of c/m = dim of x

$\frac{C}{M} = L$

or $C = [M^1 L^1 T^0]$

$\therefore$ Dim. of $C = [M^1 L^1 T^0]$ #

Q.1 [C] If $\mu = A + B/\lambda^2$ where μ = refractive index or dimensionless quantity and λ = wavelength (L) and find the dim. of 'A' & 'B'?

→ soln:

Given; $\mu - A - B/\lambda^2 = 0$

According to homogenity;

$\therefore$ dim. of $A = [M^0 L^0 T^0]$ #

Dim. of $\frac{B}{\lambda}$ = dim. of μ

or, $\frac{B}{[L^2]} = [M^0 L^0 T^0]$

or, $B = [M^0 L^2 T^0]$

$\therefore$ Dim. of $B = [M^0 L^2 T^0]$ #

Q.1[D] find dim. of x if $y = \tan(xt)$, where t is time, y is i don't know?

→ soln:

Given, $y = \tan(xt)$

dim. of $xt = [M^0 L^0 T^0]$

or, $x \times T = [M^0 L^0 T^0]$

or, $x = [M^0 L^0 T^{-1}]$ #

Q.1[E] If $P = a^{(r^2 x)}$. find dim. of x. where r is radius?

→ soln:

Given $P = a^{(r^2 x)}$

we know, power → dimension less

i.e., dim. of $r^2 x = [M^0 L^0 T^0]$

or, $x \times [L^2] = [M^0 L^0 T^0]$

or, $x = [M^0 L^{-2} T^0]$

∴ Dim. of $x = [M^0 L^{-2} T^0]$ #

Q.[2] Find the dimensional formula of potential Difference?

→ soln:

$$\text{potential Diff} = \frac{W}{Q} = \frac{F \times disp.}{I \times t} = \frac{[MLT^{-2}][L]}{A \times [T]} = [M^1 L^2 T^{-3} A^{-1}]$$

$Q = IT$

Q.[3] find the dim. formula of Resistace?

→ soln: $$\text{Resistance} = \frac{V}{I} = \frac{W}{Q \times I} = \frac{W}{I \times T \times I} = \frac{F \times disp.}{[I^2][T]} = \frac{MLT^{-2}L}{[A^2][T]}$$

$$= [M^1 L^2 T^{-3} A^{-2}]$$ #

☆ Limitation of Dimensional anlysis:

1.) It cannot give information about dimensionless constant involved in a physical relation.
2) It is very difficult to derive the dimensional relation between more than three physical quantities.
3.) It cannot be use to derive the relation involving trigonometric exponential and logarithmic functions.
4) It can't tell us whether a quantity is vector or scalar.
5.) The dimensionally correct relation may not be physically correct.

☆ Error:-

The difference between standard value and observed value of a physical quantity is called error.

i.e. Error = standard value - observed value.

Note:-

The measurement of a physical quantity is expressed as $x \pm y$ where x is observed value and y is error. Which means the standard value of this measurement lies between $(x+y)$ to $(x-y)$.

Q. The measurement of resistance of a conductor is expressed as $(10 \pm 5\%)\,\Omega$. What does it means?

Soln: The measurement of resistance of a conductor is expressed as $(10 \pm 5\%)\,\Omega$ which means $10\,\Omega$ is observed value and 5% is error, so the standard value of this resistance lies between $(10 - 0.05)\,\Omega$ to $(10 + 0.05)\,\Omega$.

☆ Types of error:-

(a) Systematic error:-

The error introduced due to the limitation of the formula used and fault in the instrument is called systematic error.

(b) Random error:-

The error introduced due to the carelessness of the experiment and unfavourable condition of the enviroment such as temperature, pressure and humidity etc is called Random error.

☆ Accurate measurement:-

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The measurement in which observed value of any physical quantity is closer to the standard value of that physical quantity is called accurate measurement.

Example :- Standard value of g in lab is

9.8 m/s². The measured value of g. is obtained as 9.67 m/s² and 9.79 m/s². The value of 9.79 m/s² is very closed to the standard value. So, it is more accurate than the value 9.67 m/s².

★ Precise measurement:-

The measurement in which the observed value of a physical quantity can be reproduced again and again by repeated experiment & procedure is called precise measurement.

If the reading are very close to each other, they are called precise reading. The thickness of a glass plate measured by spherometre are 2.43 mm, 2.44 mm, 2.42 mm and 2.44 mm. these reading are very closed to each other. So they are precise measurement.

Least Count:-

★ Significant figure:-

The meaningfull digits of a number are called significant figure. Singnificant figure depends on the least count of a measuring device.

Example:

Let us take the length of rod by a ruler for three times. Reading are obtained as 10.2 cm, 10.3 cm and 10.3 cm. So. mean length

$$= \frac{10.2 + 10.3 + 10.3}{3}$$

$$= 10.266666 \text{ cm}$$

Here all digits are not significant only first three digits are significant. The length of rod is taken as 10.2 cm or 10.3 cm #

Q. The length of a rod is exactly 1 cm. An observe records the reading as 1.0 cm, 1.00 cm and 1.000 cm which is most accurate measurement?

★⇒ Soln:

$$\text{Length} = \frac{1.0 + 1.00 + 1.000}{3} = \frac{3.000}{3} = 1.000$$

Hence, the length of rod = 1.0 or 1.00 #